AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XI

MODEL EXAMINATION PAPER 2023 AND ONWARDS

Mathematics Paper II

Time: 1 hour and 30 minutes Marks: 50

INSTRUCTIONS

Please read the following instructions carefully.

1. Check your name and school information. Sign if it is accurate.

I agree that this is my name and school. **Candidate's Signature**

RUBRIC

2. There are EIGHT questions. Answer ALL questions. Choices are specified inside the paper.

3. When answering the questions:

   Read each question carefully.
   Use a black pointer to write your answers. DO NOT write your answers in pencil.
   Use a black pencil for diagrams. DO NOT use coloured pencils.
   DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
   Complete your answer in the allocated space only. DO NOT write outside the answer box.

4. The marks for the questions are shown in brackets ( ).

5. A formulae list is provided on page 2 and 3. You may refer to it during the paper, if you wish.

6. You may use a scientific calculator if you wish.

Aga Khan University Examination Board

List of Formulae for Mathematics XI

Note:

- All symbols used in the formulae have their usual meaning.
- The same formulae will be provided in the annual and re-sit examinations.

Complex Numbers

$|z| = \sqrt{a^2 + b^2}$

Matrices and Determinants

$A_{ij} = (-1)^{i+j} M_{ij}$ $\quad AdjA = (A_{ij})^t$ $\quad A^{-1} = \frac{1}{|A|} AdjA$

Sequence & Series and Miscellaneous Series

$a_n = a_1 + (n-1)d$ $\quad A = \frac{a+b}{2}$ $\quad S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$a_n = a_1 r^{n-1}$ $\quad G = \pm\sqrt{ab}$ $\quad H = \frac{2ab}{a+b}$

$S_n = \frac{a_1(1-r^n)}{1-r}$, if $|r| < 1$ $\quad S_n = \frac{a_1(r^n - 1)}{r-1}$, if $|r| > 1$ $\quad S_\infty = \frac{a_1}{1-r}$, where $|r| < 1$

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$

Permutations, Combinations and Probability

${}^nP_r = \frac{n!}{(n-r)!}$ $\quad {}^nC_r = \frac{n!}{(n-r)!r!}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\quad P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\quad P(A \cap B) = P(A) \times P(B)$

Binomial Theorem and Mathematical Induction

$(a+x)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \binom{n}{3}a^{n-3}x^3 + \ldots + \binom{n}{n-1}a^1 x^{n-1} + x^n$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots$

$T_{r+1} = \binom{n}{r}a^{n-r}x^r$

Quadratic Equation

$x^2 - Sx + P = 0$ $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad D = b^2 - 4ac$

Introduction to Trigonometry and Trigonometric Identities

$l = r\theta$ $\quad \sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ $\quad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\quad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ $\quad \sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ $\quad \tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$ $\quad a^2 = b^2 + c^2 - 2bc\cos\alpha$ $\quad \dfrac{a-b}{a+b} = \dfrac{\tan\dfrac{\alpha-\beta}{2}}{\tan\dfrac{\alpha+\beta}{2}}$

$\cos P - \cos Q = -2\sin\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$ $\quad \sin P - \sin Q = 2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\cos P + \cos Q = 2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$ $\quad \sin P + \sin Q = 2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$ $\quad \tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

$\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$

Application of Trigonometry

$\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}ac\sin\beta = \dfrac{1}{2}ab\sin\gamma$ $\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \dfrac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$ $\quad R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma}$

$r_1 = \dfrac{\Delta}{s-a}$, $r_2 = \dfrac{\Delta}{s-b}$ and $r_3 = \dfrac{\Delta}{s-c}$ $\quad r = \dfrac{\Delta}{s}$ $\quad R = \dfrac{abc}{4\Delta}$

Graphs of Trigonometric Functions, Inverse Trigonometric Functions and Solution of Trigonometric Equations

$\sin^{-1}A \pm \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} \pm B\sqrt{1-A^2}\right)$ $\quad \cos^{-1}A \pm \cos^{-1}B = \cos^{-1}\left(AB \mp \sqrt{(1-A^2)(1-B^2)}\right)$

$\tan^{-1}A \pm \tan^{-1}B = \tan^{-1}\left(\dfrac{A \pm B}{1 \mp AB}\right)$

Q.1. (Total 4 Marks)

For the simultaneous linear equations with complex coefficients $i\,x-\sqrt{-4}\,y=\frac{8}{i}$ and $\frac{x}{i}-iy=i^3$, show that the value of y is real.

Q.2. (Total 6 Marks)

The matrix form of system of simultaneous linear equations $x+3y+3z=1,\ y+2z=2$, and $2x+y=3$ is $\begin{bmatrix}1 & 3 & 3\\ 0 & 1 & 2\\ 2 & 1 & 0\end{bmatrix}\times\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$. Find the value of the matrix $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ using matrix inversion method.

Q.3. (Total 6 Marks)

a. Convert 0.481481481481481… into an equivalent fraction. (3 Marks)

b. Find the sum of $2\times1^3+2\times2^3+2\times3^3+2\times4^3+...+2\times50^3$. (3 Marks)

Q.4. (Total 6 Marks)

a. Prove that $\binom{n}{n-1} = n$, where n is a positive integer. (2 Marks)

b. If $\binom{n}{3} = 8\binom{n-1}{n-2}$, then find the value of n. (4 Marks)

Q.5. (Total 6 Marks)

In the binomial expansion of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^n$,

a. show that the term is independent of x for $r=\frac{n}{2}$ (3 Marks)

b. hence, find the value of n and of k if the term independent of x is given by ${}^kC_3$. (3 Marks)

(**Note**: $T_{r+1} = {}^nC_r a^{n-r} b^r$ and the formula for rth term is $T_{r+1} = {}^nC_r a^{n-r} b^r$, where symbols have their usual meanings.)

(ATTEMPT EITHER PART a OR PART b OF Q.6.)

Q.6. (Total 6 Marks)

a. Find the values of x for the equation $2\left(x^2+2x\right)+\sqrt{x^2+2x+5}=0$ (6 Marks)

(**Note**: Verification not required)

b. A shopkeeper sold x cartons of dairy milk for Rs 10,500. If he had 5 more cartons, the same amount of money he would have received by selling all the cartons for Rs 50 less per carton.

With reference to the given situation,

i. state an expression for the discounted cost of each carton. (2 Marks)

ii. show that the equation represents the total cost as $(x+5)(10{,}500-50x)=10{,}500\,x$. (2 Marks)

iii. find x, which is the number of cartons. (2 Marks)

(ATTEMPT EITHER ANY TWO OF a, b AND c FOR Q.7.)

Q.7. (Total 10 Marks)

a. Show that $\sqrt{\frac{1}{r}} \times \sqrt{\frac{1}{r_1}} \times \sqrt{\frac{1}{r_2}} \times \sqrt{\frac{1}{r_3}} = \frac{1}{sr}$ (5 Marks)

b. For a real number α, $1+\cot^2(-\alpha) = \operatorname{cosec}^2(-\alpha)$. Prove this statement using Pythagoras theorem. (5 Marks)

c. If the measurements of a triangle PQR is such that $\angle P = 35.3^{o}, \angle P + \angle Q = 80.5^{o}$ and $PR = 421$ units, then find the

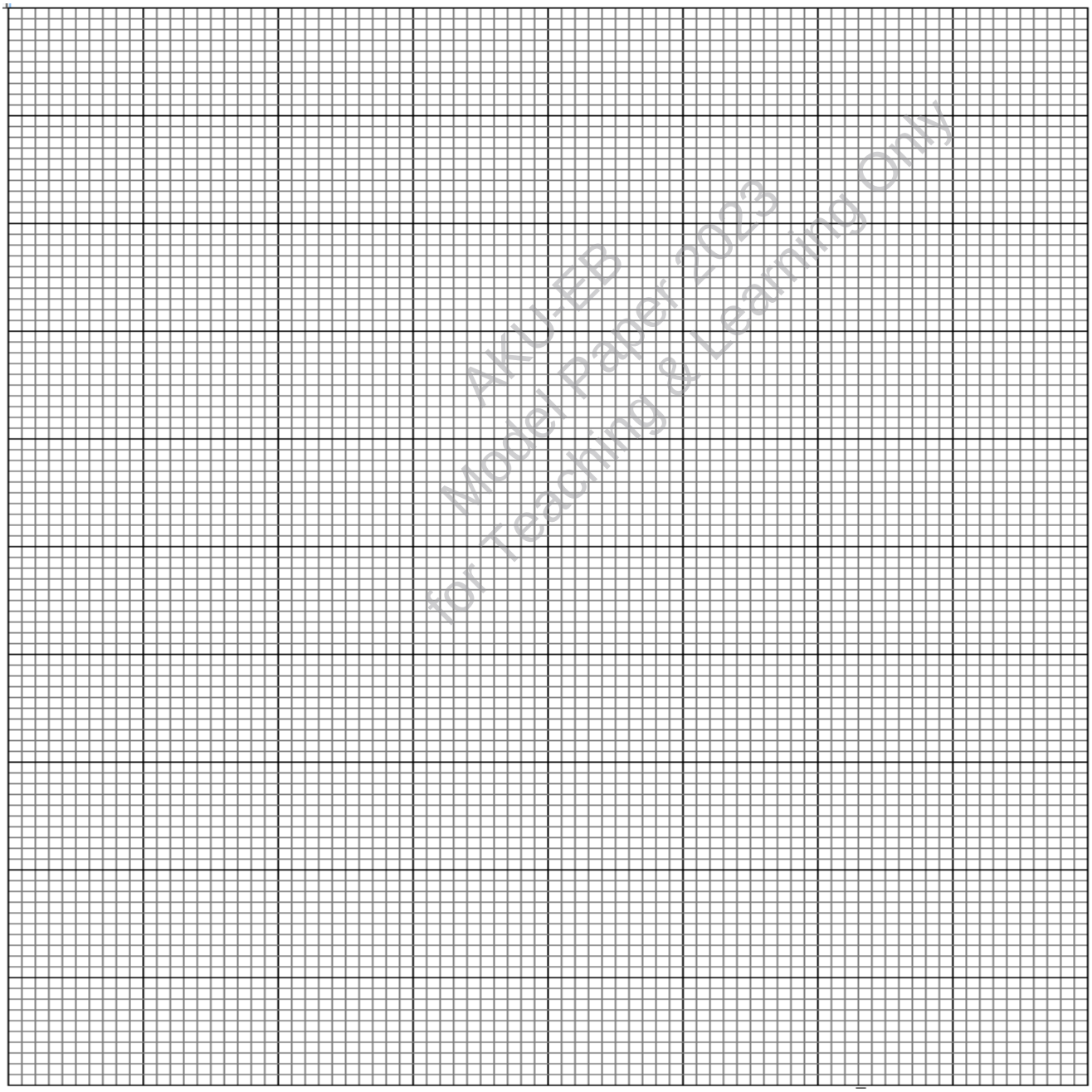
i. value of the angles $\angle Q$ and $\angle R$. (2 Marks)

ii. lengths of the remaining sides. (3 Marks)

Q.8. (Total 6 Marks)

Complete the given table and draw the graph of $y = \tan x$.

x	**0**	**30°**	**60°**	**90°**	**120°**	**150°**	**180°**	**210°**	**240°**	**270°**	**300°**	**330°**	**360°**
$y = \tan x$													



END OF PAPER

Please use this page for rough work

Please use this page for rough work

Please use this page for rough work

Please use this page for rough work