AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XI

ANNUAL EXAMINATIONS (THEORY) 2024

Mathematics Paper II

Time: 1 hour and 30 minutes Marks: 50

INSTRUCTIONS

Please read the following instructions carefully.

1. Check your name and school information. Sign if it is accurate.

I agree that this is my name and school. Candidate's Signature

RUBRIC

2. There are EIGHT questions. Answer ALL questions. Choices are specified inside the paper.

3. When answering the questions:

   Read each question carefully.
   Use a black pointer to write your answers. DO NOT write your answers in pencil.
   Use a black pencil for diagrams. DO NOT use coloured pencils.
   DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
   Complete your answer in the allocated space only. DO NOT write outside the answer box.

4. The marks for the questions are shown in brackets ( ).

5. A formulae list is provided on page 2 and 3. You may refer to it during the paper, if you wish.

6. You may use a scientific calculator if you wish.

List of Formulae

Note:

- All symbols used in the formulae have their usual meaning.

Complex Numbers

$$|z| = \sqrt{a^2 + b^2}$$

Matrices and Determinants

$$A_{ij} = (-1)^{i+j} M_{ij} \qquad AdjA = (A_{ij})^t \qquad A^{-1} = \frac{1}{|A|} AdjA$$

Sequence & Series and Miscellaneous Series

$$a_n = a + (n-1)d \qquad A = \frac{a+b}{2} \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$a_n = ar^{n-1} \qquad G = \pm\sqrt{ab} \qquad H = \frac{2ab}{a+b}$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r} \text{ if } |r| < 1 \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \text{ if } |r| > 1 \qquad S_\infty = \frac{a}{1-r}, \text{ where } |r| < 1$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Permutations, Combinations and Probability

$${}^nP_r = \frac{n!}{(n-r)!} \qquad {}^nC_r = \frac{n!}{(n-r)!r!}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad P(A/B) = \frac{P(A \cap B)}{P(B)} \qquad P(A \cap B) = P(A) \times P(B)$$

Binomial Theorem and Mathematical Induction

$$(a+x)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \binom{n}{3}a^{n-3}x^3 + \ldots + \binom{n}{n-1}a^1x^{n-1} + x^n$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots$$

$$T_{r+1} = \binom{n}{r}a^{n-r}x^r$$

Quadratic Equation

$$x^2 - Sx + P = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad D = b^2 - 4ac$$

Introduction to Trigonometry and Trigonometric Identities

$l = r\theta$ $\quad\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ $\quad \tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ $\quad \sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ $\quad \tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$ $\quad a^2 = b^2 + c^2 - 2bc\cos\alpha$ $\quad \dfrac{a-b}{a+b} = \dfrac{\tan\dfrac{\alpha-\beta}{2}}{\tan\dfrac{\alpha+\beta}{2}}$

$\cos P - \cos Q = -2\sin\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$ $\quad \sin P - \sin Q = 2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\cos P + \cos Q = 2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$ $\quad \sin P + \sin Q = 2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$ $\quad \cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$ $\quad \tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

Application of Trigonometry

$\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}ac\sin\beta = \dfrac{1}{2}ab\sin\gamma$ $\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \dfrac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$ $\quad R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma}$

$r_1 = \dfrac{\Delta}{s-a}$, $r_2 = \dfrac{\Delta}{s-b}$ and $r_3 = \dfrac{\Delta}{s-c}$ $\quad r = \dfrac{\Delta}{s}$ $\quad R = \dfrac{abc}{4\Delta}$

Inverse Trigonometric Functions and Trig. Equations

$\sin^{-1}A \pm \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} \pm B\sqrt{1-A^2}\right)$ $\quad \cos^{-1}A \pm \cos^{-1}B = \cos^{-1}\left(AB \mp \sqrt{(1-A^2)(1-B^2)}\right)$

$\tan^{-1}A \pm \tan^{-1}B = \tan^{-1}\left(\dfrac{A \pm B}{1 \mp AB}\right)$

Q.1. (Total 4 Marks)

Find the complex factors of $x^2 - 4x + 5$ by using completing square method.

Q.2. (Total 6 Marks)

Consider the given system of equations.

$$x + 2y = 5,\ x - z = -15 \text{ and } -x + 3y + 2z = 40$$

The determinant of the coefficient matrix is 1. Using Cramer's rule, find the values of x, y, and z that satisfy the given system of linear equations.

Q.3. (Total 6 Marks)

i. A city's population is growing at a rate of 2% per year. If the population was 100,000 at the start of the year, then what will be the total population in 10 years? (3 Marks)

ii. The Harmonic mean between two positive numbers, a and b, is 8.

I. Find ab. (2 Marks)

II. Hence, show that the G.M between a and b is $\pm 2\sqrt{a+b}$. (1 Mark)

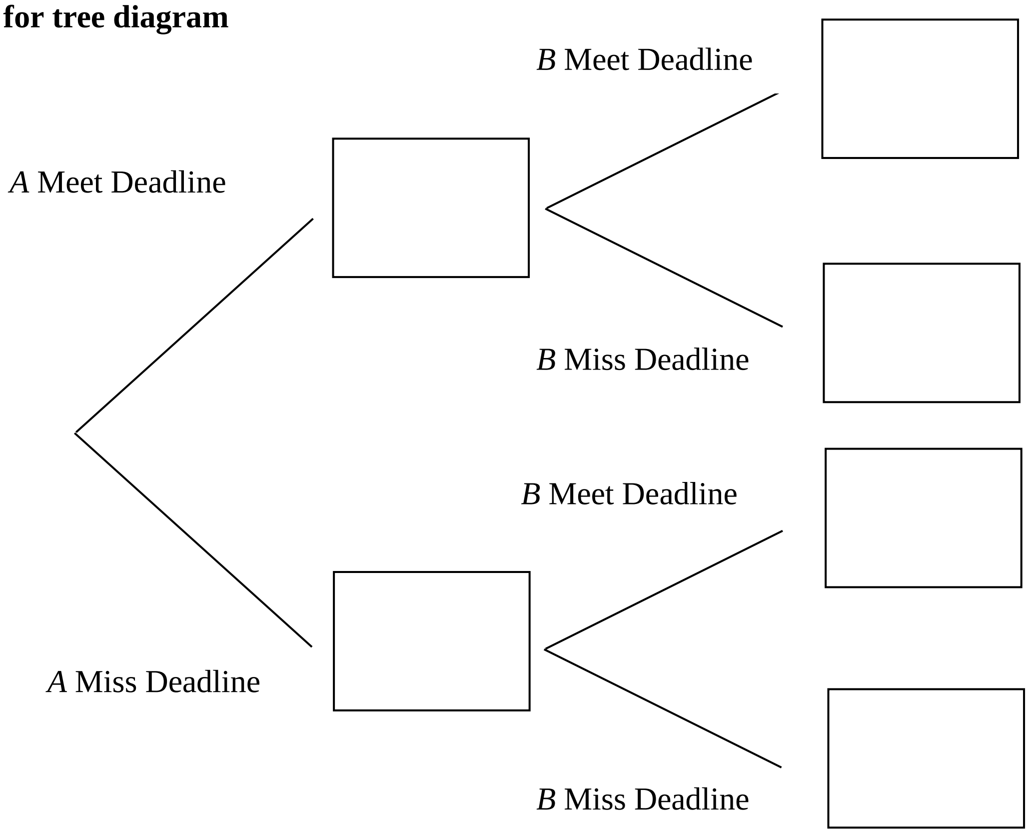
Q.4. (Total 6 Marks)

A software company has two teams, *A* and *B*, working on different modules of a project independently. There is a 20% chance that team *A* will miss a deadline, and a 30% chance that team *B* will miss a deadline.

i. Draw a tree diagram for the given situation.

ii. Use tree diagram to find the probability that one team will miss a deadline.

Space for tree diagram



Q.5. (Total 6 Marks)

Consider the expression $\sqrt[3]{k+x}$.

i. Prove that this expression can also be written as $k^{\frac{1}{3}}\left(1+\frac{x}{k}\right)^{\frac{1}{3}}$. (1 Mark)

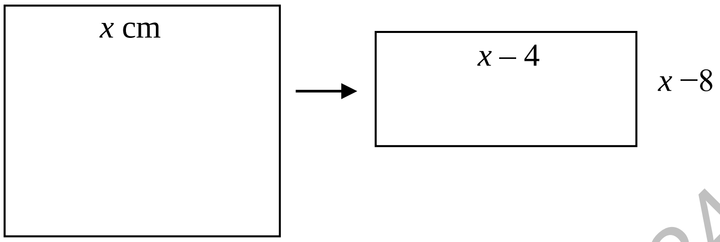
ii. Expand the expression $k^{\frac{1}{3}}\left(1+\frac{x}{k}\right)^{\frac{1}{3}}$ upto 3rd term. (2 Marks)

iii. If the coefficients of x and x^2 are equal, then prove the values of k are $-\frac{1}{3}$ and 0. (3 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.6.)

Q.6. (Total 6 Marks)

a. Consider a square of side x cm. This square is changed into a rectangle by decreasing one of its sides by 8 cm and the other by 4 cm as shown in the given figure.



NOT TO SCALE

i. Find an expression for the area of the rectangle. (1 Mark)

ii. The area of the given square is $\frac{1}{5}$ times the area of the rectangle. Show that $x^2+3x-8=0$. (3 Marks)

iii. Without using a calculator, find the length of the side of square. (2 Marks)

(**Note:** Choose the appropriate length of the square from the values found in part iii.)

(ATTEMPT EITHER PART a OR PART b OF Q.6.)

b. Solve the following system of equations. (Total 6 Marks)

$x^2 + y^2 + 2y = 16$
$3x + y = 6$

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

Q.7. (Total 10 Marks)

a. Using half angle and double angles formulae, prove that $\sin^4 x + \cos^4 x = 2\cos^4 x - 2\cos^2 x + 1$.

(**Note:** $\cos 2\theta = 2\cos^2\theta - 1$) (5 Marks)

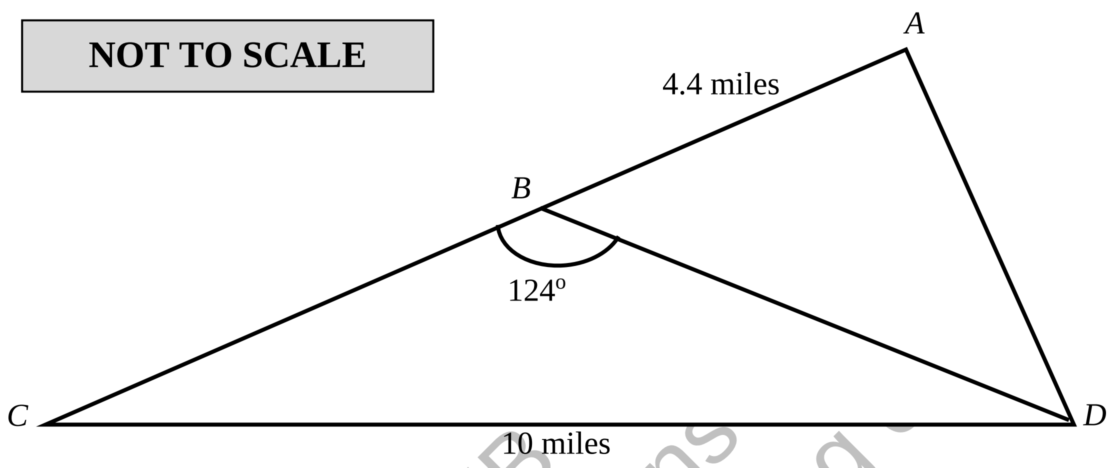
(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

b. It is given that $\cos x = \frac{2}{3}$ and $\tan y = \frac{3}{4}$.

Find the value of $\sin(x - y)$. (5 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

c. A road map shows four cities labelled *A*, *B*, *C*, and *D*. Cities *A*, *B*, and *C* lie on a straight line with distances *AB* = 4.4 miles *BD* = 6 miles, and *CD* = 10 miles and the angle *CBD* measures as 124°.



i. Find the angles *BCD* and *BDC*. (2 Marks)

ii. Find the length of *BC*. (1 Mark)

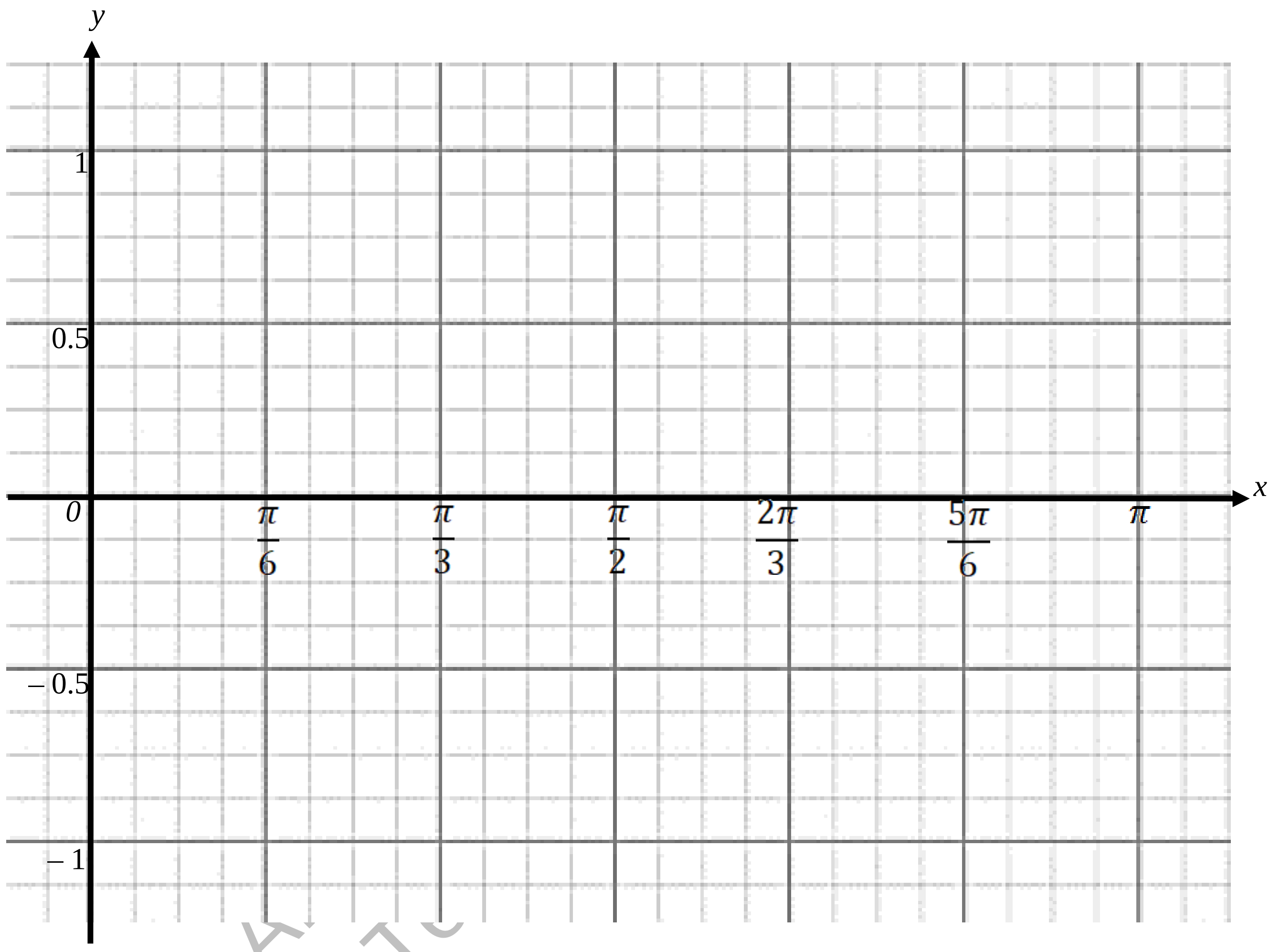
iii. Find the angle *ABD*. (1 Mark)

iv. Calculate the area of triangle ABD. (1 Mark)

Q.8. (Total 6 Marks)

Consider the functions $y = 1 - \sin x$ and $y = \cos 2x$.

i. Sketch a single graph that shows these functions for the given interval [0, π]. (4 Marks)



ii. Determine the value of k that makes the equation $\cos 2x + k^2 \sin x = 1$ true for the x-coordinates of the points where the two graphs intersect, as mentioned in part (i). (2 Marks)

END OF PAPER

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