Aga Khan University Examination Board

Notes from E-Marking Centre on HSSC-I Mathematics Examination May 2017

Introduction

This document has been produced for the teachers and candidates of Higher Secondary School Certificate (HSSC-I) Mathematics. It contains comments on candidates' responses to the 2017 HSSC-I Examination indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on students' performance on every question and *some* specific examples of students' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

General Observations

- It was noted that candidates who failed to comprehend formulae and their applications according to given situation did not score well in the examination.
- It was noted that candidates made mistakes in the formulae of trigonometry.
- It was also noted that candidates failed to comprehend the concepts of permutation, combination and row and column operations in matrices.

Detailed Comments:

Constructed Response Questions (CRQs)

Question 1:

The question was generally well-attempted..

> **Question 1:**
>
> i. Find the real and imaginary parts of the complex number $Z = \left(\dfrac{3-2i}{4-3i}\right)^2$.

Better responses exhibited that candidates correctly rationalized the denominator. Most of the candidates used the correct formulae of $(a-b)^2$ and a^2-b^2 that lead to correct answer. In some responses it was noted that candidates first applied the formula of $(a-b)^2$ followed by the simplification process and finally rationalised the denominator to separate the real and imaginary parts of the given complex number.

Example:

$$\frac{3^2-2(3)(2i)+(2i)^2}{4^2-2(4)(3i)+(3i)^2}$$

$$\frac{9-12i+4i^2}{16-24i+9i^2}$$

$$\frac{9-4-12i}{16-9-24i}$$

$$\frac{5-12i}{7-24i}\times\frac{7+24i}{7+24i}$$

$$\frac{5(7+24i)-12i(7+24i)}{(7)^2-(24i)^2}$$

$$\frac{35+120i-84i-288i^2}{49+576}$$

$$\frac{35+288+36i}{625}$$

$$\frac{323+36i}{625}$$

$$\frac{323}{625}+\frac{36}{625}i$$

Real part: $\frac{323}{625}$

Imaginary Part: $\frac{36}{625}$

Weaker responses showed that in part I, candidates made mistakes in application of formulae of $(a-b)^2$ and a^2-b^2 and basic arithmetical operations on complex numbers. Consequently, candidates were not able to separate the real and imaginary parts of the complex number.

Example 1:

$$Z = \left(\frac{3-2i}{4-3i}\right)^2$$

$$Z = \frac{(3-2i)^2}{(4-3i)^2}$$

$$Z = \frac{3^2 - 2(3)(2i) + (2i)^2}{4^2 - 2(4)(3i) + (3i)^2} \qquad \therefore (a+b)^2 = a^2+2ab+b^2 \quad \therefore (a-b)^2 = a^2-2ab+b^2$$

$$Z = \frac{9 - 12i + 4i^2}{16 - 24i + 9i^2} \qquad \therefore i^2 = -1$$

$$Z = \frac{9 - 12i - 4}{16 - 24i - 9}$$

$$Z = \frac{5}{7} - \frac{12i}{24i}$$

$\frac{5}{7}$ = real

$\frac{\cancel{12}i}{\cancel{24}i} = \frac{1i}{2}$ = Imeginery

Example 2:

$$z = \frac{(3-2i)^2}{(4-3i)^2}$$

$$\frac{(9)^2 - 2(3)(2i) + (2i)^2}{(4)^2 - 2(4)(3i) + (3i)^2}$$

$$\frac{81 - 12i + 4(i^2)}{16 - 24i + 9(i^2)}$$

$$\frac{81 - 12i - 4}{16 - 24i - 9}$$

$$\frac{77 - 12i}{7 - 24i}$$

$$11\ \frac{\cancel{77}}{\cancel{7}} - \frac{\cancel{12}^{1}}{\cancel{24}_{2}} i$$

$$11 - \frac{1}{2} i$$

11 (real part) $-\frac{1}{2}$ (imaginary part)

Question 1:

ii. For $Z_1 = 13 - 12i$, prove that $|Z_1| = |\overline{Z}_1|$.

Better responses exhibited that candidates found the correct conjugate of $Z_1 = 13 - 12i$, and applied formula of modulus correctly to prove $|Z_1| = |\overline{Z}_1|$.

Example:

$z_1 = 13 - 12i$	$\bar{z} = 13 + 12i$
$\lvert z\rvert = \sqrt{a^2+b^2}$	$\lvert\bar{z}\rvert = \sqrt{a^2+b^2}$
$\lvert z\rvert = \sqrt{13^2+(-12)^2}$	$\lvert\bar{z}\rvert = \sqrt{13^2+(12)^2}$
$\lvert z\rvert = \sqrt{313}$	$\lvert\bar{z}\rvert = \sqrt{313}$
$z\rvert = 17.69$	$\lvert\bar{z}\rvert = 17.69$

Weaker responses displayed that candidates were confused about the formula of modulus and conjugate of the complex number and, therefore, failed to prove the required result.

Example 1:

$|Z_1| = |13 - 12i| \Rightarrow 13 - 12i$ and $-(13 - 12i)$

$|\bar{Z}_1| = |13 + 12i| \Rightarrow 13 + 12i$ and $-(13 + 12i)$

Example 2:

$\lvert Z\rvert = \sqrt{(x)^2-(y)^2}$	$\bar{Z} = 13 + 12i$
$\lvert Z_1\rvert = \sqrt{(13)^2-(12)^2}$	$\lvert\bar{Z}_1\rvert = \sqrt{x^2-y^2}$
$\lvert Z_1\rvert . \sqrt{169-144}$	$\lvert\bar{Z}_1\rvert = \sqrt{(13)^2-(12)^2}$
$\lvert Z_1\rvert = \sqrt{25}$	$\lvert\bar{Z}_1\rvert = \sqrt{169-144}$
$\lvert Z_1\rvert = \pm 5$	$\lvert\bar{Z}_1\rvert = \sqrt{25} = \pm 5$
	so $\lvert Z_1\rvert = \lvert\bar{Z}_1\rvert$

Question 2:

This question was attempted well by most of the candidates.

Question 2:

i. Find the matrix A in terms of a, if $\begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix} = 2A - \begin{bmatrix} 0 & -a & -a \\ -a & 0 & -a \\ -a & -a & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$.

Also, identify the type of matrix A.

Better responses showed that the candidates skilfully applied the concept of multiplication, addition and scalar multiplication of matrices to find the value of unknown matrix X from the given matrix equations.

Example:

Sol

$$\begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix} = 2A - \begin{bmatrix} 0+0+0 & 0+(-a)+0 & 0+0+(-a) \\ -a+0+0 & 0+0+0 & 0+0+(-a) \\ -a+0+0 & 0+(-a)+0 & 0+0+0 \end{bmatrix}$$

$$\begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix} = 2A - \begin{bmatrix} 0 & -a & -a \\ -a & 0 & -a \\ -a & -a & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix} + \begin{bmatrix} 0 & -a & -a \\ -a & 0 & -a \\ -a & -a & 0 \end{bmatrix} = 2A$$

$$2A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \qquad A = \begin{bmatrix} a/2 & 0 & 0 \\ 0 & a/2 & 0 \\ 0 & 0 & a/2 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$ It is diagonal as well as scalar matrix.

Weaker responses showed that candidates mainly failed to perform matrix multiplication $\begin{bmatrix} 0 & -a & -a \\ -a & 0 & -a \\ -a & -a & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$ and showed misconceptions about addition of matrices and failed to find the matrix X.

Example 1:

$$\begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix} = 2A - \begin{bmatrix} 0 & -a & -a \\ -a & 0 & -a \\ -a & -a & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix} = 2A - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix} = 2A \implies A = \begin{bmatrix} \frac{a}{2} & \frac{a}{2} & \frac{a}{2} \\ \frac{a}{2} & \frac{a}{2} & \frac{a}{2} \\ a/2 & a/2 & a/2 \end{bmatrix}$$

A is a Square matrix

Example 2:

$$2A - \begin{bmatrix} 0 & -a & -a \\ -a & 0 & -a \\ -a & -a & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$2A - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2a & 2a & 2a \\ 2a & 2a & 2a \\ 2a & 2a & 2a \end{bmatrix}$$

Question 2ii:

Without expansion, show that $\begin{vmatrix} 3 & 6 & 9 \\ 2 & 5 & 8 \\ -1 & 3 & 7 \end{vmatrix} = 0.$

Better responses exhibited that candidates applied the multiple methods of solution and aptly applied row operations or column operations to make two rows or two columns identical and to prove that the given determinant $\begin{vmatrix} 3 & 6 & 9 \\ 2 & 5 & 8 \\ -1 & 3 & 7 \end{vmatrix}$ is equal to zero.

Example 1:

By subtracting $C_3 - C_2$ and then adding the result in C_1 we get

$$\begin{vmatrix} 3+(9-6) & 6 & 9 \\ 2+(8-5) & 5 & 8 \\ -1+(7-3) & 3 & 7 \end{vmatrix} = \begin{vmatrix} 3+3 & 6 & 9 \\ 2+3 & 5 & 8 \\ -1+4 & 3 & 7 \end{vmatrix} = \begin{vmatrix} 6 & 6 & 9 \\ 5 & 5 & 8 \\ 3 & 3 & 7 \end{vmatrix}$$

Since Two coloums are identical so the determinant $= 0$

Weaker responses reflected lack of understanding of properties of determinants. Few candidates used correct properties but made simple errors in addition and subtraction of corresponding elements of a row or a column of the determinant and failed to prove the required result. Though, it was mentioned in the question that the requirement is to show that the given determinant is zero without expansion but few candidates used the expansion of the determinant to solve the question.

Example 1:

taking 3 common from R_1 $R_1 - R_2$

$$3\begin{vmatrix}1 & 2 & 3\\ 2 & 5 & 8\\ -1 & 3 & 7\end{vmatrix} \qquad \begin{vmatrix}1 & 1 & 1\\ 2 & 5 & 8\\ -1 & 3 & 7\end{vmatrix}$$

$R_2 - R_1$

$$\begin{vmatrix}0 & 0 & 1\\ 3 & 3 & 8\\ -2 & -4 & 7\end{vmatrix}$$

when two identical columns so ~~there~~ determinant equal to zero.

Example 2:

Subtract 1 from Row 1

$$\begin{vmatrix}3-1 & 6-1 & 9-1\\ 2 & 5 & 8\\ -1 & 3 & 7\end{vmatrix}$$

$$\begin{vmatrix}2 & 5 & 8\\ 2 & 5 & 8\\ -1 & 3 & 7\end{vmatrix} = 0$$, When Two rows are same Determinant $= 0$

Question 3:

If the 4th term of an arithmetic progression is 2 and the 9th term is $\frac{9}{2}$, then find the first term and the common difference. Also, find the 100th term of the progression.

This was a generally a well-attempted question.

Better responses exhibited that candidates applied correct formula of general term of an arithmetic sequence and successfully found the value of a and d by developing and solving two equations, i.e. $2 = a + 3d$ and $\frac{9}{2} = a + 8d$. Finally, candidates were able to find the 100th term of the progression.

Example:

$a_4 = 2$ $\quad a_1 + 3d = 2$ —(1)

$a_9 = \frac{9}{2}$ $\quad a_1 + 8d = \frac{9}{2}$ —(2) subtract from eq. 1

$\cancel{a_1} + 3d = 2$

$-\cancel{a_1} \oplus 8d = -\frac{9}{2}$

$-5d = \frac{-9}{2} + 2$

$-5d = \frac{-9+4}{2}$

$\cancel{+5}d = \frac{\cancel{+5}}{2}$ $\quad d = \frac{1}{2}$ Now put d in eq. (1)

$a_1 + 3d = 2$

$a_1 + 3\left(\frac{1}{2}\right) = 2$

$a_1 + \frac{3}{2} = 2$

$a_1 = \frac{1}{2}$

Now find 100 term

$a_n = a_1 + (n-1)d$

$a_{100} = \frac{1}{2} + (100-1)d$

$a_{100} = \frac{1}{2} + 99\left(\frac{1}{2}\right)$

$\frac{1}{2} + \frac{99}{2}$

$\frac{1+99}{2} = \frac{\cancel{100}^{50}}{\cancel{2}}$

$a_{100} = 50$

Weaker responses reflected that candidates either failed to write the correct formula or made mistakes in application of the formula. Few common mistakes have been mentioned below.:

$$a_n = a(n-1)d$$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Some weaker responses applied the concept of ratio which was not applicable in the given situation.

Example 1:

Put $n=1$ 1, 2, 3, 4, 5

$a_n = a + (n-1)d$

$a_1 = a + (1-1)d$

$a_1 = a + 0d = a$

$a_2 = a + 2(2)$

$= a + 4$

$S_{100} = \frac{100}{2}(2a + (n-1).$

Example 2:

Term	Value
4th	2
1st	x

$4x = 2$

$x = 2/4$

$x = 0.5$

Common difference $= 1 - 0.5$

$= 0.5$

100th term $= 100 \times 0.5$

$= 50$

Example 3:

[illegible] ,

$T_4 = 2$ & $T_9 = 9/2$ $T_1 = ?$ $d = ?$

$T_{100} = ?$

$d = 9$

$n = 1$

$T_9 = a(n-1)d$

$T_1 = 1(1-1)9$

$T_1 = 1 \times 9$

$\boxed{T_1 = 9}$

$T_{100} = a(n-1)d$

$T_{100} = 2(\cancel{100}-1)9$

$T_{100} = 2(99)9$

$\boxed{T_{100} = 1782}$

So,

* ANSWER $\boxed{d = 9}$ $\boxed{T_1 = 9}$ $\boxed{T_{100} = 1782}$

Question 4:

This question offered a choice between part **a**. and **b**. Most candidates performed well in this question. Majority of the students attempted part a.

Question 4a:
Without using calculator, convert 0.481481481481481…. into an equivalent common fraction.

Better responses indicated that most of the candidates were able to write the given number as 0.481481481481481…= 0.481+0.000481+0.000000481+0.000000000481+… and then successfully identified the given series as infinite geometric series and applied formula correctly to find the required equivalent common fraction.

Example:

$0 + (0.481 + 0.000481 + 0.000000481 \ldots)$

$a = 0.481$

$r = 0.000481/0.481 = 0.001$

$S_\infty = a/1-r = 0.481/1-0.001$

$= \dfrac{0.481}{0.999}$

$= 13/27$ Ans.

Weaker responses reflected various types of confusions to convert the given decimal number to equivalent common fraction. Few approximated it to third decimal place and then removed the decimal by writing 1000 in the denominator as cited in Example 1. In few other weaker responses it is noted that candidates were able to identify the correct geometric series but failed to apply the formula correctly. Other weaker responses revealed that candidates wrote the correct formula but failed to identify correct value of *a* and *r*, hence failed to fulfil the requirement of the question.

Example 1:

(a) 0.481481481

$= 0.481$

$= \dfrac{0.481}{1000} = \dfrac{481}{1000}$ (Ans)

Example 2:

consider G.P = $0.481 + 0.000481 + 0.000000481 + \ldots\ldots$

$r = \frac{1}{1000}$ or 1×10^{-3}

$S_\infty = \frac{a_1}{r}$

$= \frac{0.481}{1 \times 10^{-3}}$

$= 481$

Example 3:

$[0.481 + 0.0481 + 0.00481 \ldots]$

$r = \frac{0.048}{0.481}$

$r = 0.1$

$S_n = \frac{a_1}{1 - r}$

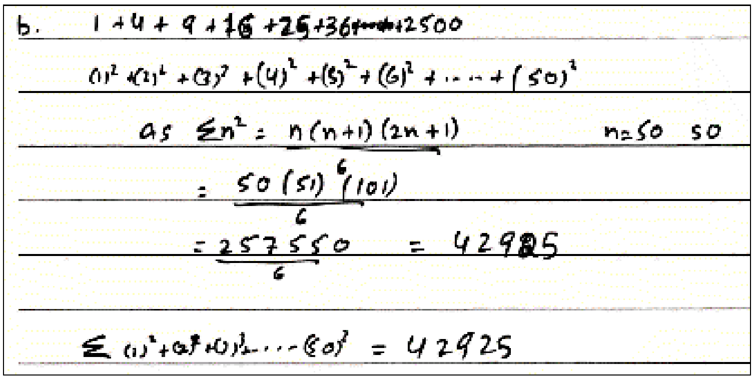
$S_n = \frac{0.481}{1 - 0.1} \Rightarrow \frac{0.481}{0.9} \Rightarrow \frac{0481}{1000} \cdot \frac{10}{9} = \frac{481}{900}$ required fraction

Question 4b:

Find the sum of the series 1 + 4 + 9 + 16 + 25 + 36 + …. + 2500.

Better responses correctly converted the given series to sum of square of first fifty integers as $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \ldots + 50^2$ and applied the correct formula of $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$ to get the required sum.

Example:


b. $1+4+9+16+25+36+\dots+2500$

$(1)^2+(2)^2+(3)^2+(4)^2+(5)^2+(6)^2+\dots+(50)^2$

as $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$ n=50 50

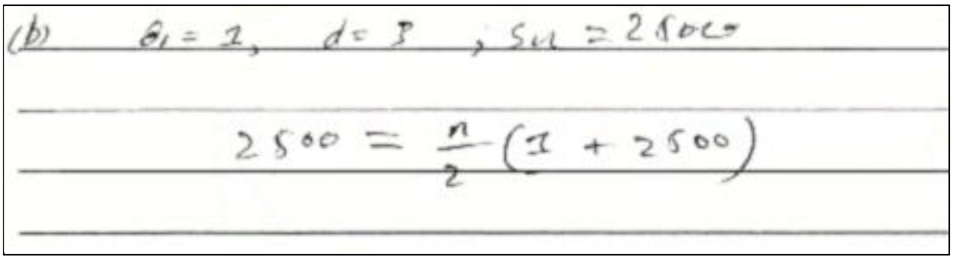
$= \frac{50(51)(101)}{6}$

$= \frac{257550}{6} = 42925$

$\sum (1)^2+(2)^2+(3)^2\dots(50)^2 = 42925$

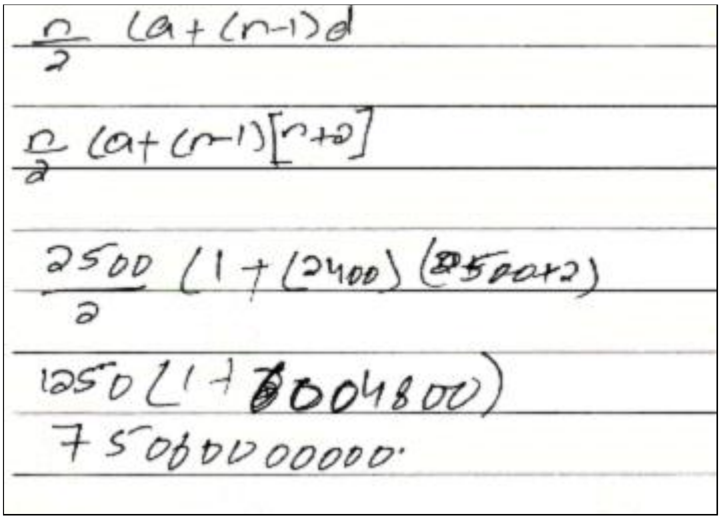
Weaker responses suggested that candidates were unable to understand the question. They used the formula of arithmetic sequence, arithmetic series, geometric sequence or geometric series. This was not required to solve the given question.

Example 1:


(b) $a_1 = 1$, $d = 3$, $S_n = 2500$

$2500 = \frac{n}{2}(1 + 2500)$

Example 2:


$\frac{n}{2}(a+(n-1)d$

$\frac{n}{2}(a+(n-1)[n+2]$

$\frac{2500}{2}(1+(2400)(2500+2)$

$1250(1+004800)$

7500000000

Question 5:

This question offered a choice between part **a** and **b**. Most candidates did not perform well in this question. Majority of the students chose to attempt part b.

Question 5a:

Prove that $\binom{n}{3}+\binom{n}{2}=\binom{n+1}{3}$, where n is a positive integer.

Better responses exhibited that candidates correctly applied the formula to expand $\binom{n}{3}$, $\binom{n}{2}$ and $\binom{n+1}{3}$ to desired form or terms and skilfully took the L.C.M. and simplified the terms to produce right hand side and able to accomplish prove of the required result.

In other better responses candidates took L.H.S and applied formula of combination and simplified the result, then they took the R.H.S and applied the formula and done cancellation to attain term which is equal to L.H.S. and completed the proof.

Example 1:

LHS

$$\frac{n!}{3!(n-3)!} + \frac{n!}{2!(n-2)!}$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}} + \frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}}$$

$$\frac{n(n-1)(n-2)}{3!} + \frac{n(n-1)}{2!}$$

$$\frac{4n(n-1)(n-2) + 6n(n-1)}{12}$$

$$\frac{\cancel{4}\,n\left[2(n^2-n-2n+2)+3n-3\right]}{6}$$

$$\frac{n}{6}\left(2n^2-6n+4+3n-3\right)$$

$$\frac{n}{6}\left(2n^2-3n+1\right)$$

$$\frac{n}{6}\left(2n^2-2n-n+1\right)$$

$$\frac{n}{6}\left(2n(n-1)-1(n-1)\right)$$

$$\frac{n}{6}\left(2n-1\right)(n-1)$$

$$(2n-1)\times\frac{n(n-1)}{3!}$$

$$\frac{n!}{3!(n-3)!} + \frac{n!}{2!(n-2)!}$$

$$\frac{2!\,n!\,(n-2)}{3!\times 2!(n-3)!(n-2)} + \frac{3!\,n!}{2!\times 3!(n-2)!}$$

$$\frac{2n!(n-2) + 6n!}{12(n-2)!}$$

$$\frac{\cancel{2}n!\left((n-2)+3\right)}{\cancel{12}(n-2)!}$$

$$\frac{n!(n+1)}{6(n-2)!}$$

$$\frac{(n+1)(n!)}{3!(n-2)!}$$

$$\frac{(n+1)!}{3!(n-3+1)!}$$

$$\frac{(n+1)!}{3!((n+1)-3)!}$$

$$\therefore \binom{n+1}{3} = RHS \text{ Proved}$$

Example 2:

$$^{n}C_3 + {}^{n}C_2 = {}^{n+1}C_3$$

$$L.H.S = \frac{n!}{3!(n-3)!} + \frac{n!}{2!(n-2)!}$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}} + \frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}}$$

$$\frac{n(n-1)(n-2)}{6} + \frac{n(n-1)}{2} = \frac{n(n-1)(n-2)+3n(n-1)}{6}$$

$$\frac{n(n-1)}{6}\left[n-2+3\right] = \frac{n(n-1)(n+1)}{6}$$

$$R.H.S = \frac{(n+1)!}{(3!)(n+1-3)!} = \frac{(n+1)(n)(n-1)\cancel{(n-2)!}}{3!\,\cancel{(n-2)!}}$$

$$= \frac{n(n-1)(n+1)}{6}$$

R.H.S = L.H.S proved.

Weaker responses reflected weak concept of combination and application of its formula. They also made mistakes in the simplification process. In a few weaker responses, candidates applied the technique of mathematical induction and were able to prove it for n =1 but failed to proceed for n= k and n= k+1 as cited in example 1.

Example 1:

Now let us prove that this statement is true for n=1

$\binom{1}{3}+\binom{1}{2}=\binom{1+1}{3}$

$\frac{1!}{(1-3)!3!}+\frac{1!}{(1-2)!2!}=\frac{(1+1)!}{(2-3)!3!}$

$\frac{1}{12}+\frac{1}{4}=\frac{2}{6}$

$\frac{1}{3}=\frac{1}{3}$ ∴ Hence this statement is true for n=1

Now we assume that this statement is true for n=K

${}^{K}C_3+{}^{K}C_2={}^{K+1}C_3$

Now let us prove this statement is true for n=K+1

${}^{K+1}C_3+{}^{K+1}C_2={}^{K+2}C_3$

this statement is true because

${}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r$

or $\binom{K+1}{3}+\binom{K+1}{2}=\binom{K+2}{3}$

n

${}^{n}C_3+{}^{n}C_2={}^{n+1}C_3$

$\frac{n!}{(n-3)!3!}+\frac{n!}{(n-2)!2!}=\frac{(n+1)!}{(n+1-3)!3!}$

$\frac{n!}{(n-3)!3!}+\frac{n!}{(n-2)!2!}=\frac{n+1}{(n-1}$

Example 2:

$\binom{n}{3}+\binom{n}{2}=\binom{n+1}{3}$

I n $\binom{n+1}{3}$

$=\binom{1+1}{2}=\binom{3}{2}$

C⇒II n = K

$\binom{K+1}{3}$

C⇒III (K+1)(K+1

Example 3:

$$\frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r)!\,r!} = \frac{(n+1)!}{(n-r)!\,r!}$$

$$\frac{n!}{(n-3)!\,3!} + \frac{n!}{(n-2)!\,2!} = \frac{(n+1)!}{(n-3)!\,3!}$$

$n=1$

$$\frac{1!}{(1-3)!\,3!} + \frac{1!}{(1-2)!\,2!} = \frac{(1+1)!}{(1-3)!\,3!}$$

$$\frac{1!}{(-2)!\,3!} + \frac{1!}{(-1)!\,2!} = \frac{(2)!}{(-2)!\,3!}$$

$$\frac{1}{-12} + \frac{1}{-2} = \frac{2}{-12}$$

$$\frac{1}{-12} + \frac{2}{12} = \frac{1}{2}$$

$$\frac{1}{12}$$

Question 5b:

A non-transparent jar contains tickets numbered from 1 to 50. If two dice are rolled simultaneously and ticket is drawn from the jar, then answer the following questions.

i. Find the number of sample points in the sample space.

ii. Are rolling a dice and drawing a ticket from the jar independent or dependent events? Justify your answer.

iii. What is the probability of both dice showing the same number and ticket showing a number greater than 45? Also write all possible outcomes of the event.

iv. Find the probability of the event that the sum on both dice is 9 and the ticket shows odd numbers.

Better responses exhibited good understanding of probability theory. Candidates understood the question well and were able to find the correct number of the sample points in the given situation. The sample points were further used to find the probability of the given situations in the next part of the question. The better responses also indicated that candidates were clear about the concept of dependent and independent events and possible outcomes of an event.

Example:

i. Find the number of sample points in the sample space. (1 Mark)

$= 6^2 \times 50 = 1800$ sample points.

ii. Are rolling a dice and drawing a ticket from the jar independent or dependent events? Justify your answer. (2 Marks)

independent event.

Because rolling a die not influence the drawing of ticket from the jar and that is independent event.

iii. What is the probability of both dice showing the same number and ticket showing a number greater than 45? Also write all possible outcomes of the event. (2 Marks)

n(A) = {(1,1,46)(1,1,47)(1,1,48)(1,1,49)(1,1,50), (2,2,46)(2,2,47)
(2,2,48)(2,2,49)(2,2,50)(3,3,46)(3,3,47)(3,3,48)
(3,3,49)(3,3,50)(4,4,46)(4,4,47)(4,4,48)(4,4,49)
(4,4,50)(5,5,46)(5,5,47)(5,5,48)(5,5,49)(5,5,50)
(6,6,46)(6,6,47)(6,6,48)(6,6,49)(6,6,50)}

$P(A) = \frac{n(A)}{n(S)} = \frac{30}{1800} = \frac{1}{60} = 0.01666$

iv. Find the probability of the event that the sum on both dice is 9 and the ticket shows odd numbers. (1 Mark)

$n(B) = 100$

$n(S) = 1800$

$P(B) = \frac{n(B)}{n(S)} = \frac{100}{1800} = \frac{1}{18}$.

Weaker responses reflected that students were unable to find the number of sample points of the sample space for the given situation. Few candidates used addition model instead of multiplication model and few wrote incorrect number of sample points directly without using addition or multiplication rule. Weaker responses also exhibited that students were not clear about the difference of independent and dependent events. They also failed to find the probability for the situation given in the question. It was generally not a well attempted question, which is indicative of the fact is that probability theory needs more attention from teachers and students.

Example 1:

i. Find the number of sample points in the sample space. (1 Mark)

$n(S) = \{1764\}$.

ii. Are rolling a dice and drawing a ticket from the jar independent or dependent events? Justify your answer. (2 Marks)

Drawing a ticket from a jar is dependent.
but rolling of dice are independent events.
Drawn ticket depends upon the ticket jar.

iii. What is the probability of both dice showing the same number and ticket showing a number greater than 45? Also write all possible outcomes of the event. (2 Marks)

let A be event for both dice showing same no.

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$.

$P(A) = n(A)/n(S) = \frac{6}{36} = 1$

let B be the event for ticket showing greater number.

$B = \{46, 47, 48, 49\}$.

$B = \frac{4}{49}$.

$P(A \cup B) = P(A) + P(B)$

$= \frac{6}{36} + \frac{4}{49}$

$P(A \cup B) = \frac{294 + 144}{1764}$.

$P(A \cup B) = 0.24 = 73/294$.

iv. Find the probability of the event that the sum on both dice is 9 and the ticket shows odd numbers. (1 Mark)

A be the event for sum of dice is 9, $A = \{(4,5), (3,6), (5,4), (6,3)\} = 4/36 = 1/9$.

B be the event for ticket show odd no $B = \{1, 3, 5, 7, 11, 13, (15), (17), (19), (21, \ldots 2n+1)\} = \frac{16}{49} = 0.326$

Example 2:

i. Find the number of sample points in the sample space (1 Mark)

$P(A \cap B) = \frac{1}{50} \times \frac{2}{6} = \frac{2}{300} = \frac{1}{150}$

ii Are rolling a dice and drawing a ticket from the jar independent or dependent events? Justify your answer. (2 Marks)

Independent because rolling a dice ~~es~~ does not depend on the previous dice number or drawing a ticket is also independent because it does not change any previous event :

iii. What is the probability of both dice showing the same number and ticket showing a number greater than 45? Also write all possible outcomes of the event. (2 Marks)

DICE : {(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)} O(A):6 , O(S):36

{(46)(47)(48)(49)(50)} O(B): 5 O(S): 50

$P(A) = \frac{O(A)}{O(S)} = \frac{6}{36} = \frac{1}{6}$

$P(B) = \frac{O(B)}{O(S)} = \frac{5}{50} = \frac{1}{10}$

iv. Find the probability of the event that the sum on both dice is 9 and the ticket shows odd numbers. (1 Mark)

~~DICE = {(4,5)(5,4)} TICKET=~~ Dice: {(3,6)(6,3)(4,5)(5,4)}

TICKET = {5,7,9} : $P(A) = \frac{1}{9}$, $P(B) = \frac{1}{12}$ $P(A) + P(B) = \frac{7}{36}$.

Question 6:

i. Prove using mathematical induction that $5+9+13+17+.......+4n+1=n(2n+3)$ is true for all positive integral values of *n.* Support your working with necessary statements

The question was generally well-attempted.

Better responses indicated that candidates systematically followed the steps of mathematical induction. They proved that the given statement is true for n = 1 by substituting n=1and to prove that the statement is true for n=k+1, they added $4k$ +5 to both sides and broke the middle term to prove the required result for all positive integral values of n .

Example:

let tak $n=1$

$5 = 1(2(1)+3)$

$5=5$ True for $n=1$

*Assumption

lets assume that n is treure for k.

$5+9+13 \ldots\ldots 4k+1 = k(2k+3)$

* Check for $k=k+1$

$4(k+1)+1 = 4k+4+1$

$=4k+5$

Adding $4k+5$ on both sides

$5+9+13 \ldots 4k+1+4k+5 = k(2k+3)+4k+5$

$= 2k^2+3k+4k+5$

$= 2k^2+7k+5$

$2k^2+2k+5k+5$

$2k(k+1)+5(k+1)$

$(2k+5)(k+1)$

Weaker responses displayed that candidates were able to prove the result for $n = 1$, but failed to add the correct term to prove the truth of the statement for n=K+1 and failed to factorise the term on the right hand side to prove the given statement.

Example 1:

$n = 1$

$4(1) + 1 = 1(2(1)+3)$

$4 + 1 = 2 + 3$

$5 = 5 \quad 1 = 1 \quad n = 1$ is true for all positive integers

$n = (k+1)$

$4(k+1)+1 = (k+1)\,[2(k+1)+3)$

$4k + 4 + 1 = (k+1)(2k+2+3)$

$4k + 5 = (k+1)(2k+5)$

$4k+5 = 2k^2 + 5k + 2k + 5$

$4k+5 = 2k^2 + 7k + 5$

$4k+5 = 2k - 2k - 5k + 5$

$4k + 5 = 2k(k)-1) + 5k(-1+1)$

$4k + 5 = -3k$

$-4k - 3k = +5 \quad -7k = 5$ not true for all positive integers.

Example 2:

Let Sn be the statement $S_n: 5+9+13+17+ \dots +4n+1 = n(2n+3)$.

put $n=1$. - prove for n.

$4n+1 = n(2n+3)$

$4+1 = 1(2+3)$

$5=5 \rightarrow$ it is true for $n=1$

we assume that $S(k)$ will be true for all Z^+

$- - - - +4k+1 = k(2k+3)$ — ①.

we shall prove for $(k+1)$ true for all Z^+.

$\dots + 4(k+1+1) = (k+1)(2(k+1)+3)$ — ②.

Adding $4(k+1+1)$ in eq① on b/s.

$+ \dots + 4(k+1) + (4(k+1+1)) = k(2k+3) + (4(k+1+1))$.

$= 2k^2 + 3k + 4k + 8$.

$= 2k^2 + 7k + 8$.

=

$k(2k+3) + 4((k+1)+1)$.

$(k+1)\quad(2(k+1)+3)$.

hence it is true for all.

+ve Z^+.

Question 6:

ii. Find the third term in the expansion of $(2a-3b)^{10}$.

Better responses exhibited that most of the candidates correctly used the general term formula for finding a specific term in the binomial expansion. They correctly identified the value of *a*, *b*, *n* and *r* and substituted in the formula. Finally, they simplified correctly to find the third term of the given binomial expansion. Some candidates expanded $(2a-3b)^{10}$ by applying binomial theorem to third term to accomplish the task given in the question.

Example 1:

$n=10 \quad r=2 \quad a=2a \quad b=-3b$

$T_3 = {}^{10}C_2\,(2a)^{10-2}\,(-3b)^2 \qquad \therefore T_{r+1} = {}^{n}C_r\, a^{n-r}\, b^r$

$T_3 = 45 \cdot 256a^8 \cdot 9b^2$

Example 2:

$(2a)^{10} + (10)(2a)^{10-1}(-3b)^1 + \frac{10(10-1)}{2!}(2a)^{10-2}(-3b)^2$

$1024a^{10} + 10(2a)^9(-3b) + 45 \times {}^{2!}(2a)^8(-3b)^2$

$1024a^{10} + (-18\,2\,320$

$\cancel{1024a^{10}} - \cancel{60a^9}$

$1024a^{10} - 15360a^9b + \cancel{34560}\ 103680a^8b^2$

Third term is $103680a^8b^2$

Weaker responses showed that candidates used incorrect general term formula for finding specific term in the binomial expansion. Those who used the correct formula got stuck in the simplification and were not able to find the exponent of *n* and *r*. It was also observed that candidates who applied binomial expansion failed to apply the formula.

Example:

$(2a-3b)^{10} = 2a^{10} - 10(2a)^9(3b) + 45(2a)^8(3b)^2 + \cdots$

$(2a+3b)^{10} = 2a^{10} + \ldots$

the third term is $45(2a)^8(3b)^2$.

Question 7:

This question offered a choice between part **a** and **b**. Candidates chose to attempt part **a** more than part b.

Question 7a:
Find the solution set of the equation $(x+1)(x+2)(x+3)(x+4)=3$.

Better responses showed that candidates re-arranged the given equation $(x+1)(x+2)(x+3)(x+4)=3$ as $(x+1)(x+4)(x+2)(x+3)=3$ and then multiplied to get $(x^2+5x+4)(x^2+5x+6)=3$. They made supposition $y=x^2+5x$ to reduce the given quatirc equation to quadratic equation. To solve the newly obtained quadratic equation, mostly candidates applied the method of breaking of middle term to get the two values of *y*. Consequently, they solved the resulting two quadratic equations by using quadratic formula to get the values of variable *x* and wrote the required solution set.

Example 1:

a. $(x+1)(x+2)(x+3)(x+4)=3$

$2+3=5=4+1$

$(x+2)(x+3)(x+4)(x+1)=3$

$(x^2+3x+2x+6)(x^2+x+4x+4)=3$

$(x^2+5x+6)(x^2+5x+4)=3$

let $t=x^2+5x$

$(t+6)(t+4)=3$

$t^2+4t+6t+24=3$

$t^2+10t+21=0$

$a=1,\ b=10,\ c=21$

$t=\dfrac{-10\pm\sqrt{10^2-4(1)(21)}}{2(1)}$

$t=\dfrac{-10\pm\sqrt{100-84}}{2}$

$t=\dfrac{-10\pm\sqrt{16}}{2}$

$t=\dfrac{-10+4}{2}=\dfrac{-6}{2}$, $\boxed{t=-3}$ $\qquad$ $t=\dfrac{-10-4}{2}$, $t=\dfrac{-14}{2}$, $\boxed{t=-7}$

Now,

$t=-3$

$x^2+5x=-3$

$x^2+5x+3=0$

$x=\dfrac{-5\pm\sqrt{25-4(3)(1)}}{2(1)}$

$x=\dfrac{-5\pm\sqrt{13}}{2}$

$t=-7$

$x^2+5x+7=0$

$x=\dfrac{-5\pm\sqrt{5^2-4(7)(1)}}{2(1)}$

$x=\dfrac{-5\pm\sqrt{-3}}{2}$

$\therefore S.S=\left\{\dfrac{-5\pm\sqrt{13}}{2},\ \dfrac{-5\pm\sqrt{-3}}{2}\right\}$

Ans.

Example 2:

$(x+1)(x+4)(x+2)(x+3)-3=0.$

$[x^2+4x+x+4][x^2+3x+2x+6]-3.$

$[x^2+5x+4][x^2+5x+6]-3.$

$x^2+5x=y.$

$(y+4)(y+6)=3.$

$y^2+6y+4y+24-3.$

$y^2+10y+21.$

$y^2+7y+3y+21$

$y(y+7)+3(y+7)$

$(y+7)(y+3)$

$y=-7 \quad y=-3.$

$x^2+5x=-7.$

$x^2+5x+7=0.$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{(-5) \pm \sqrt{(5)^2-4(1)(7)}}{2}$

$\frac{-5 \pm \sqrt{25-28}}{2}$

$\frac{-5 \pm \sqrt{3}i}{2}.$

$x=\frac{-5+\sqrt{3}i}{2}, \frac{-5-\sqrt{3}i}{2}$

$x^2+5x=-3.$

$x^2+5x+3=0.$

$\frac{-5 \pm \sqrt{(1)^2-4(1)(3)}}{2}$

$\frac{-5 \pm \sqrt{13}}{2}.$

$x=\left\{\frac{-5\pm\sqrt{3}i}{2}\right\}\left\{\frac{-5\pm\sqrt{13}}{2}\right\}$

Weaker responses reflected that candidates assumed each factor given in the equation equal to 3 i.e. $(x+1)=3, (x+4)=3, (x+2)=3,$ or $(x+3)=3$. This was the most common error noted in the weaker responses. Other common mistakes were of re-arrangement of factors of the given equation, mistakes in taking common and simple arithmetic errors which resulted in the loss of marks.

Example 1:

a. $(x+1)(x+2)(x+3)(x+4)=3$

$\cancel{(x^2+2x+x+2)(x^2+4x+3x+12)=3}$

$\cancel{(x^2+3x+2)(x^2+7x+12)=3}$

$x+1=3, \quad x+2=3, \quad x+3=3, \quad x+4=3$

$x=3-1, \quad x=3-2, \quad x=3-3, \quad x=3-4$

$x=2, \quad x=1, \quad x=0, \quad x=-1$

$S.S=\{2, 1, 0, -1\}$

Example 2:

$[(x+1)(x+4)][(x+2)(x+3)]=3$

$[x^2+4x+x+4][x^2+3x+2x+6]=3$

$[x^2+5x+4][x^2+5x+6]=3$

$x^4+5x^3+6x^2+5x^3+25x^2+30x+4x^2+20x+24=3$

$x^4+10x^3+35x^2+50x \quad 3-24$

$5x(2x^3+7x^2+10)=-21$

$5x=-21, \quad 2x^3+7x^2+10=-21$

$\boxed{x=-\frac{21}{5}}, \quad x(2x^2+7x+10)=-21$

$\boxed{x=-21}, \quad 2x^2+7x+10=-21$

$2x^2+7x+10+21=0$

$2x^2+7x+31=0$

$\boxed{x=\frac{-7+\sqrt{199}\,i}{4}}$

and

$\boxed{x=\frac{-7-\sqrt{199}\,i}{4}}$

Question 7b:

i. If the roots of equation $x^2 + 2x - 15 = 0$ are α and β, then find the equation whose roots are $\alpha + \beta$ and $\frac{1}{\alpha} + \frac{1}{\beta}$.

ii. Apply synthetic division to find quotient and remainder when $x^5 - 3x^3 + x + 7$ is divided by $x - 1$.

Generally, it was not a well attempted question and mostly responses showed that part i of the question was not attempted by the candidates at all and most scripts were found blank.

Better responses of part i showed that candidates found the correct sum and product of the roots of the given equation and then of the required equation and finally substituted the calculated values in the formula $x^2 - Sx + p = 0$ to find the given equation.

In part ii, candidates first wrote the coefficient of x^5, x^4, x^3, x^2, x and constant correctly. They applied the process of synthetic division of the given algebraic expression $x^5 - 3x^3 + x + 7$ by $x - 1$ and were able to find the quotient and remainder of the given algebraic expression.

Example 1:

(b)(i) $x^2+2x-15=0$

$a=1,\ b=2,\ c=-15$

$\alpha = \frac{-2+\sqrt{4-4(1)(-15)}}{2}$, $\beta = \frac{2-\sqrt{4^2-4(1)(-15)}}{2a}$

$\alpha = \frac{-2+\sqrt{64}}{2}$, $\beta = \frac{-2-\sqrt{64}}{2}$

$\alpha = \frac{6}{2}$ $\boxed{\alpha=3}$, $\boxed{\beta=-5}$

the otherf equation whose roots are

$a = \alpha+\beta = 3-5 = \boxed{-2}$ & $b = \frac{1}{3} + \frac{1}{-5}$

$\boxed{b = \frac{2}{15}}$

$S = a+b = -2+\frac{2}{15}$

$= \frac{-30+2}{15} = \boxed{\frac{-28}{15}}$

$P = ab = \frac{-2(2)}{15} = \boxed{\frac{-4}{15}}$

we know that,

$x^2 - Sx + P = 0$

$x^2 - \left(\frac{-28}{15}\right)x + \left(\frac{-4}{15}\right) = 0$

$x^2 + \frac{28x}{15} - \frac{4}{15} = 0$

Answer.

(ii) synthetic division method.

$x^5 - 3x^3 + x + 7$, $x = \boxed{1}$.

$x^5 + 0x^4 - 3x^3 + 0x^2 + x + 7$

1	1	0	-3	0	1	7	
	↓	1	1	-2	-2	-1	
	1	1	-2	-2	-1	$\boxed{6}$	– R.

$\underbrace{1x^4 + 1x^3 - 2x^2 - 2x - 1 = 0}_{\text{quotient}}$.

$6 \rightarrow$ Remainder

Example 2:

by $x-1$.

(b)(i) $\alpha+\beta = -2 \qquad \alpha\beta = -15$

$\alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta} \qquad \alpha+\beta+\frac{\alpha+\beta}{\alpha\beta} = -2-\frac{2}{-15}$

$= \frac{30-2}{-15} = \frac{-28}{15}$

$(\alpha+\beta)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) \qquad (\alpha+\beta)\left(\frac{\alpha+\beta}{\alpha\beta}\right) = (-2)\frac{(-2)}{-15} \quad \frac{4}{-15} =$

equation is $x^2+\frac{28}{15}x-\frac{4}{15}=0$

(ii) $x = 1$

	1	0	-3	0	1	7	
1		1	1	-2	-2	-1	
	1	1	-2	-2	-1	6	

Remainder = 6

Quotient = $x^4+x^3-2x^2-2x-1$

Weaker *responses* showed that in part i, the candidates failed to understand the question. They were not able to find the sum and product of the roots for the required equation. Therefore, most of the scripts were left blank. This question was parallel in difficulty level to the questions given in the recommended textbook of the syllabus and hence better performance was expected.

In part ii, weaker responses showed that candidates failed to write the coefficient of missing terms x^4 and x^2 as zero. They also showed mistakes in the process of synthetic division like addition of numbers. Moreover, They failed to write correct quotient and remainder after division process.

Example 1:

ii. $x-1=0 \qquad x=1$

$P(x) = x^5 + -3x^3 + x + 7$

1	1	-3	1	7
	0	1	-2	-1
	1	-2	-1	-6

$x^2-2x-1-6=6$

$x^2-2x-5=0$

Example 2:

(11) $n^2 - 3n^3 + n + 7$ is divisble by $n-1$

$n-1 = a-1$

$a=1$	1	1	-3	1	7
			1	-2	1
		1	-2	1	$\boxed{6}$ — is remender.

the quotient is $n^3 - 2n + 1$ and remeinder is zero

Question 8a:

i. Verify that $\dfrac{(\sin^2\theta + \cos^2\theta)(1+\cot^2\theta)}{(1+\tan^2\theta)} = \cot^2\theta.$

ii. The terminal ray of θ lies in the fourth quadrant and $\sin\theta = -\dfrac{4}{7}$. Without using the calculator, find the value of $\cot\theta$.

Better responses showed that for part i, candidates used the left hand side and applied the correct trigonometric identities aptly to prove it to the other side. Some candidates converted $\tan^2\theta$ and $\cot^2\theta$ terms of $\sin\theta$ and $\cos\theta$ to verify the given trigonometric equation.

Example 1:

$\dfrac{(\sin^2\theta + \cos^2\theta)(1+\cot^2\theta)}{(1+\tan^2\theta)}$ $\cancel{= \cot^2\theta}$

$\dfrac{(1)(\cancel{cosec^2\theta})}{sec^2\theta}$ $\quad \dfrac{(1)\ (1+\cot^2\theta)}{1+\tan^2\theta}$

$cosec^2\theta \div sec^2\theta \quad 1+\dfrac{\cos^2\theta}{\sin^2\theta} \div 1+\dfrac{\sin^2\theta}{\cos^2\theta}$

$\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta} \div \dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$

$\dfrac{1}{\sin^2\theta} \div \dfrac{1}{\cos^2\theta} \Rightarrow \dfrac{1}{\sin^2\theta} \times \cos^2\theta$

$\dfrac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$ proved

Example 2:

L.H.S $\frac{(\sin^2\theta + \cos^2\theta)(1+\cot^2\theta)}{1+\tan^2\theta} = \frac{(1)\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

$\frac{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} = \frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta}}$

$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$ Hence proved.

Example 3:

$\frac{(\sin^2\theta+\cos^2\theta)(1+\cot^2\theta)}{(1+\tan^2\theta)}$

$\because \sin^2\theta+\cos^2\theta=1$

$(1)(1+\cot^2\theta)/(1+\tan^2\theta)$

$1+\cot^2\theta/(1+\tan^2\theta)$

$1+\cos^2\theta/\sin^2\theta/1+\sin^2\theta/\cos^2\theta$

$\sin^2\theta+\cos^2\theta/\sin^2\theta$

$\cos^2\theta+\sin^2\theta/\cos^2\theta$

$1/\sin^2\theta/1/\cos^2\theta$

$\frac{1}{\sin^2\theta} \times \cos^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta}$

$\because \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$

So; $\boxed{\cot^2\theta = R.H.S}$

Answer

Weaker responses reflected that in part i, candidates applied wrong trigonometric identities or they failed to follow hierarchy of arithmetic operations in the process of simplification.

Example 1:

$\Rightarrow \frac{(1)+ +\cot^2\theta}{1+\tan^2\theta}$

$\Rightarrow \frac{1+\cot^2\theta}{1+\frac{1}{\cot\theta}}$

$\Rightarrow \frac{\cot^2\theta+1}{\cot^2\theta}$

$\Rightarrow \frac{\cancel{1}+\cancel{1}-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \frac{-\tan^2\theta}{1} - 1 \Rightarrow -1-\tan^2\theta \Rightarrow 1+\tan^2\theta$

$\Rightarrow \boxed{\cot^2\theta} \Rightarrow R.H.S$

Hence proved.

Example 2:

i. Verify that $\frac{(\sin^2\theta+\cos^2\theta)(1+\cot^2\theta)}{(1+\tan^2\theta)}=\cot^2\theta.$

$(1)\ (1+\cot^2\theta)$ $\quad \sin^2\theta+\cos^2\theta=1$

Question 8a:

ii. The terminal ray of θ lies in the fourth quadrant and $\sin\theta=-\frac{4}{7}$. Without using the calculator, find the value of $\cot\theta$.

Better responses displayed that candidates found the value of $\cos\theta$ by applying correct trigonometric identity i.e. $\cos\theta=\sqrt{1-\sin^2\theta}$ and with correct sign as per given quadrant and successfully found the value of $\cot\theta$.

Example:

$\cos\theta=\sqrt{1-\sin^2\theta} \qquad (\cos^2\theta=1-\sin^2\theta)$

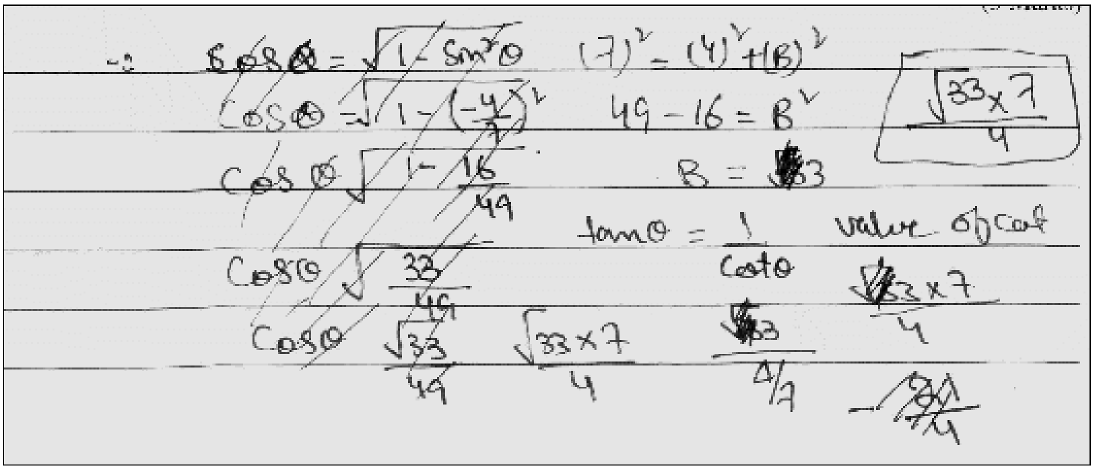
$\Rightarrow \cos\theta=\sqrt{1-\left(-\frac{4}{7}\right)^2}$

$\Rightarrow \cos\theta=\sqrt{1-\frac{16}{49}} \Rightarrow \sqrt{\frac{49-16}{49}} \Rightarrow \sqrt{\frac{33}{49}}=\frac{\sqrt{33}}{7}$

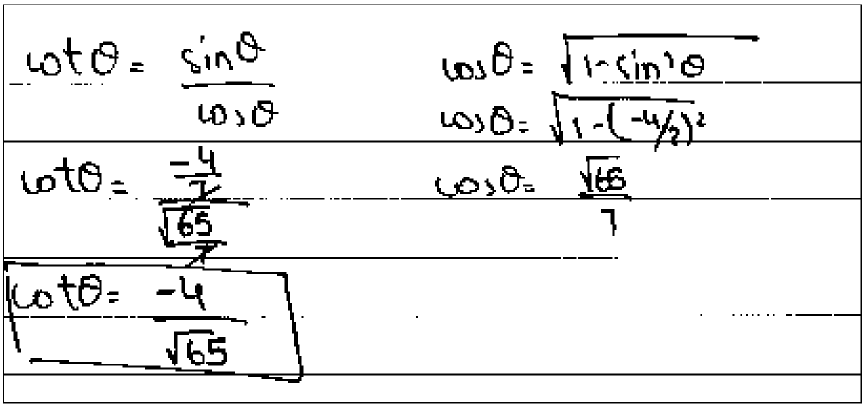
Now, $\cot\theta=\frac{\cos\theta}{\sin\theta}=\frac{\sqrt{33}}{7}\div\frac{-4}{7}\Rightarrow\frac{\sqrt{33}}{7}\times\frac{7}{-4}\ \boxed{=-\frac{\sqrt{33}}{4}}$ Ans

Weaker responses applied wrong trigonometric formula or incorrectly used Pythagorean's theorem to find the value of $\cot\theta$. In other responses, the wrong selection of sign was also evident.

Example 1:



Example 2:



Question 8b:

i. Apply formulae of $\sin(\alpha-\beta)$ and $\cos(\alpha-\beta)$ to deduce that $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$.

Better responses of part i showed that candidates used the formula of $\sin(\alpha-\beta)$ and $\cos(\alpha-\beta)$, after writing $\tan(\alpha-\beta)=\dfrac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$, and deduced the required formula. They skilfully took the L.C.M. and made correct cancellation during the process of deduction.

Example:

(4 Marks)

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$

$$\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

divide up and down by $\cos\alpha \cos\beta$

$$= \frac{\dfrac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} - \dfrac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} \qquad \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

$$\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$ Hence proved

Weaker responses showed that in part i, candidates lost marks mainly because of the incorrect use of formula, i.e. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ and $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ or failed in correct cancellation of the terms.

Example 1:

$$\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \tan(\alpha-\beta).$$

$$= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta.} = \tan(\alpha-\beta).$$

Example 2:

$$\tan\alpha-\beta = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$= \frac{\sin\alpha(1-\cancel{\sin\beta}) - \cos\alpha\sin\cancel{\beta}}{\cos\alpha(1-\cancel{\sin\beta}) + \sin\alpha\cancel{\sin\beta}}$$

$$= \frac{\sin\alpha - \cos\alpha}{1 + \cos\alpha + \sin\alpha} = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \text{Pd}$$

Question 8b:

ii. Verify that $\dfrac{\sin 3\alpha + \sin \alpha}{\cos 3\alpha - \cos \alpha} = -\cot \alpha$.

Better responses of part ii, exhibited correct use of trigonometric formulae which led to prove the correct result as required in the given question.

Example:

from L.H.S.

$\dfrac{\sin 3\alpha + \sin\alpha}{\cos 3\alpha - \cos\alpha}$ $\quad \because \sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$

$\cos P - \cos Q = -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$

$= \dfrac{2\sin\left(\frac{3\alpha+\alpha}{2}\right)\cos\left(\frac{3\alpha-\alpha}{2}\right)}{-2\sin\left(\frac{3\alpha+\alpha}{2}\right)\sin\left(\frac{3\alpha-\alpha}{2}\right)}$

$= \dfrac{\cancel{2}\,\cancel{\sin 2\alpha}\,\cos\alpha}{-\cancel{2}\,\cancel{\sin 2\alpha}\,\sin\alpha} = -\dfrac{\cos\alpha}{\sin\alpha} = -\cot\alpha$

(hence proved)

Weaker responses of part ii, exhibited that candidates used incorrect trigonometric formulae or failed to verify the required result.

Example 1:

$\dfrac{\sin(2\alpha + \alpha) + \sin\alpha}{\cos(2\alpha + \alpha) - \cos\alpha}$

$\cos(2\alpha+\alpha)$ ~~...~~ $4\cos\beta - 3\cos 2\beta$

$\sin(2\alpha+\alpha) = 3\sin^2\alpha - 4\sin\alpha$

$\dfrac{3\sin^2\alpha - 4\sin\alpha + \sin\alpha}{\text{~~...~~} - \cos\alpha}$

$4\cos\beta - 3\cos^2\beta$

Example 2:

$$= \frac{\sin 4\alpha}{\cos 2\alpha} \quad = \frac{\sin 2\alpha + \sin 2\alpha}{\cos 2\alpha}$$

$$= \frac{2\sin\alpha\cos\alpha + 2\sin\alpha\cos\alpha}{1-2\sin^2\alpha}$$

$$= \frac{4\sin\alpha\cos\alpha}{1-2\sin^2\alpha}$$

Question 9:

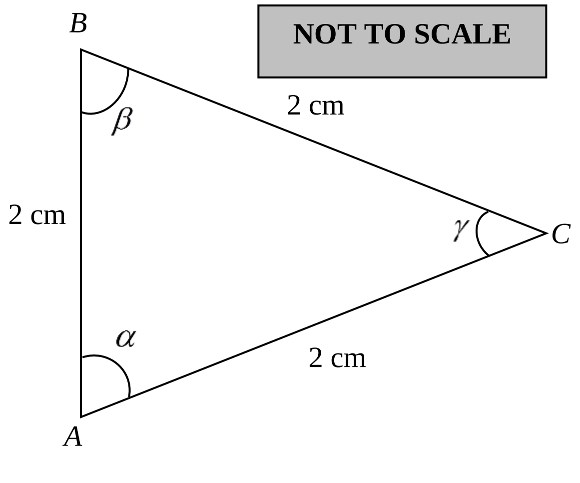
This was not a well attempted question. This question offered a choice between part **a** and **b**. Candidates chose to attempt both parts equally. However, candidates performed better in part **a** as compared to part **b**.

Question 9a:

In the given equilateral triangle *ABC*, the length of each side is 2 cm. For the given triangle *ABC*,

i. prove that $r_1 : R : r = 3 : 2 : 1$, where r_1 is the radius of the escribed circle opposite to the vertex A, R is the circum-radius and r is the in-radius.

ii. find the value of $\sin\alpha$ where α is an angle associated with vertex *A*.



Better responses showed that in part i, the candidates used the correct formulae for finding semi-perimeter, area of the triangle *ABC*, circum-radius, in-radius and escribed radius. They wrote their values as a ratio and multiplied by $\sqrt{3}$ to prove the result $r_1 : R : r = 3 : 2 : 1$.

In part ii, there were multiple options available to find the value of $\sin\alpha$. Candidates applied the correct formulae and got the value of $\sin\alpha$.

Example:

(i)

$2s = a+b+c$

$s = \frac{2+2+2}{2} = 3$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{3(3-2)(3-2)(3-2)} = \sqrt{3}$

$r_1 = \frac{\Delta}{s-a}$

$= \frac{\sqrt{3}}{3-2} = \sqrt{3}$

$r = \frac{\Delta}{s}$

$= \frac{\sqrt{3}}{3}$

$R = \frac{abc}{4\Delta}$

$= \frac{(2)(2)(2)}{4(\sqrt{3})} = \frac{2\sqrt{3}}{3}$

$r_1 : R : r$

$\sqrt{3} : \frac{2\sqrt{3}}{3} : \frac{\sqrt{3}}{3}$

$\div$ by $\sqrt{3}/3$

$r_1 : R : r$

$3 : 2 : 1$

(ii) $\Delta = \frac{1}{2} bc \sin\alpha$

$\sqrt{3} = \frac{1}{2}(2)(2)\sin\alpha$

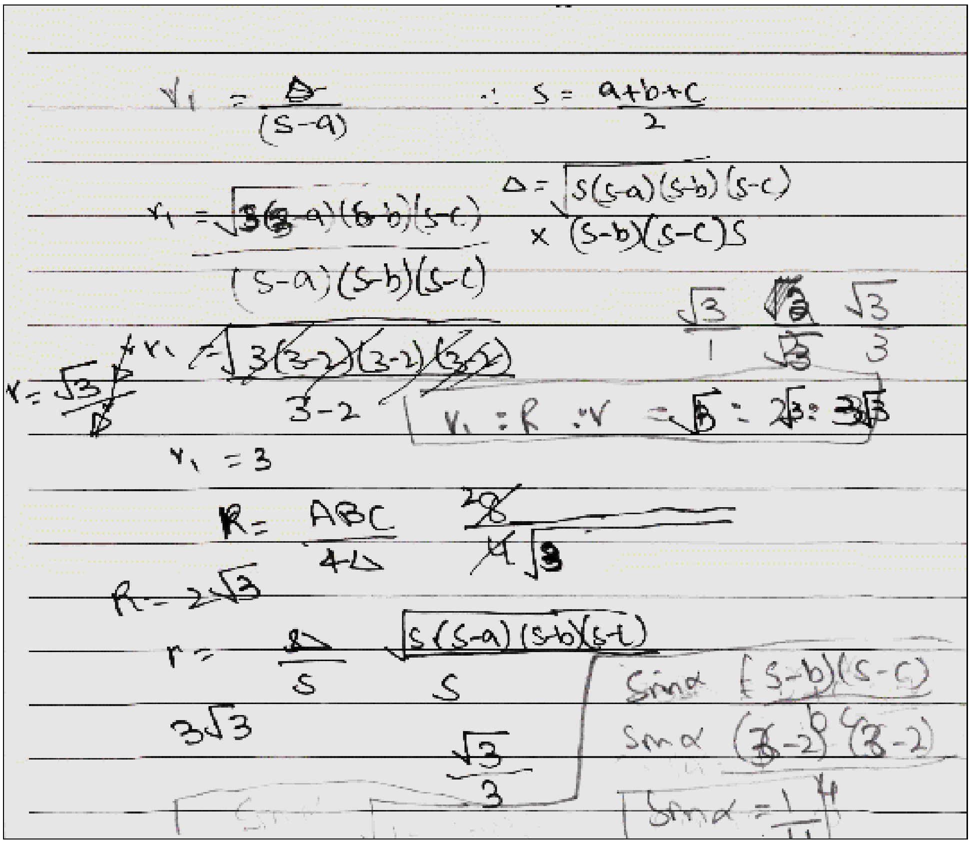
$\sqrt{3} = 2(\sin\alpha)$

$\sin\alpha = \frac{\sqrt{3}}{2}$

Weaker responses showed that most of the candidates were able to find the values of semi-perimeter, area of the triangle *ABC*, circum-radius, in-radius and escribed radius. However, they made errors in choosing which formula to use, hence, it resulted in a variety of mistakes in their solutions. Such candidates were unable to prove the required result. Few common errors have been presented in the following examples.

In part ii the wrong selections of formula like $\sin\alpha = \frac{(s-b)\times(s-c)}{bc}$ or $\sin\alpha = \frac{\sin\alpha}{\Delta}$ and mistakes in applications were noted, which resulted in wrong answer.

Example 1:



$r_1 = \frac{\Delta}{(s-a)}$ $\therefore s = \frac{a+b+c}{2}$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$r_1 = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{(s-a)(s-b)(s-c)}$ $\times (s-b)(s-c)s$

$r_1 = \frac{\sqrt{3(3-2)(3-2)(3-2)}}{3-2}$

$r = \frac{\sqrt{3}}{\Delta}$

$\frac{\sqrt{3}}{1}$ $\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{\sqrt{3}}{3}$

$\boxed{r_1 : R : r = \sqrt{3} : 2\sqrt{3} : 3\sqrt{3}}$

$r_1 = 3$

$R = \frac{ABC}{4\Delta}$ $\frac{8}{4\sqrt{3}}$

$R = 2\sqrt{3}$

$r = \frac{\Delta}{s}$ $\frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$

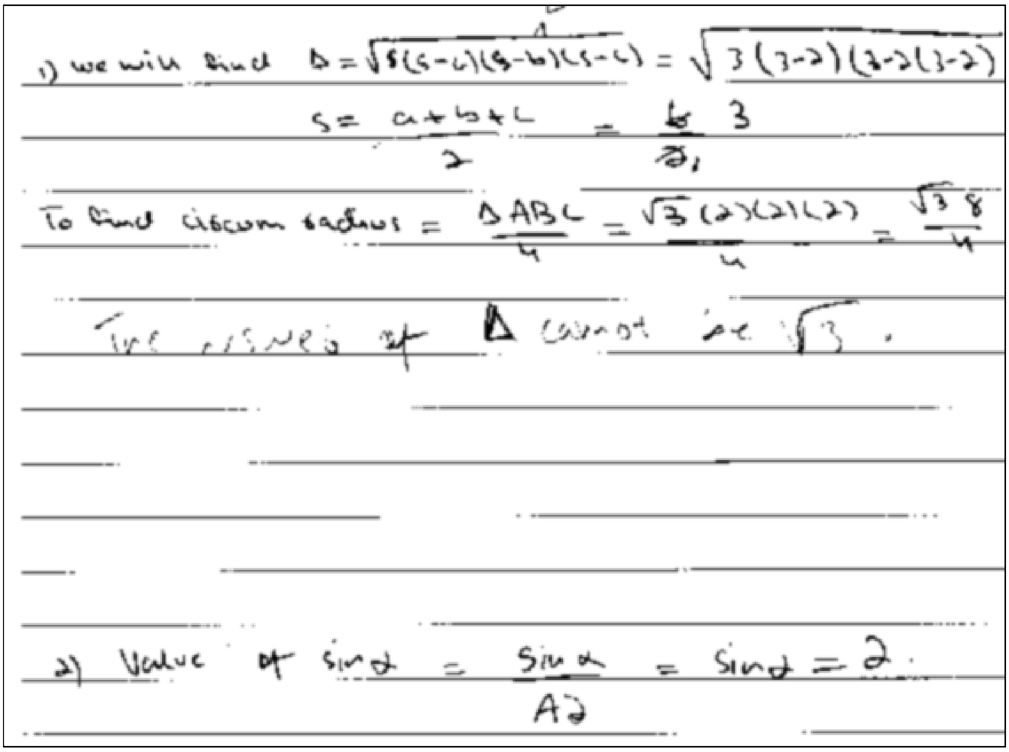
$3\sqrt{3}$ $\frac{\sqrt{3}}{3}$

$\sin\alpha \; (s-b)(s-c)$

$\sin\alpha \; (3-2)(3-2)$

$\sin\alpha = \frac{1}{4}$

Example 2:



1) we will find $\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{3(3-2)(3-2)(3-2)}$

$s = \frac{a+b+c}{2} = \frac{6}{2} = 3$

To find circum radius $= \frac{\Delta ABC}{4} = \frac{\sqrt{3}(2)(2)(2)}{4} = \frac{\sqrt{3}\,8}{4}$

The answer of Δ cannot be $\sqrt{3}$.

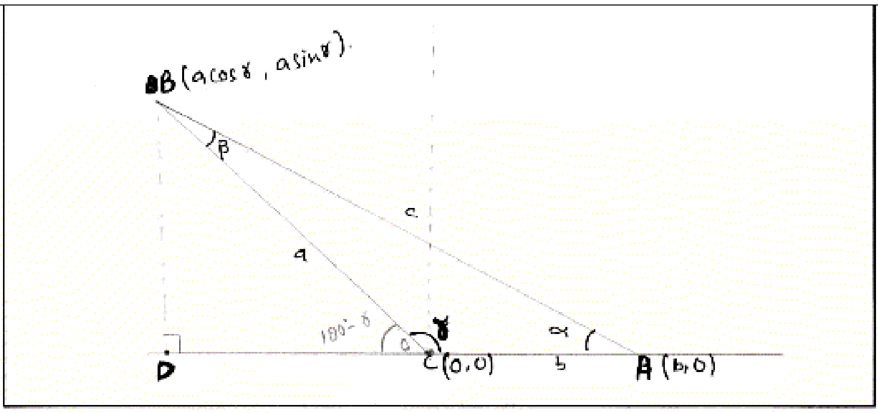
2) Value of $\sin\alpha = \frac{\sin\alpha}{A2} = \sin\alpha = 2$.

Question 9b:

i. Prove the law of cosines, i.e. $c^2 = a^2 + b^2 - 2ab\cos\gamma$ with the help of a suitable diagram.

Better responses exhibited that the candidates correctly constructed the triangle *ABC* by keeping vertex *C* at the origin and were able to find the coordinates of *A* and *B* of the triangle. Then, they applied distance formula correctly to get $c = \sqrt{a^2 + b^2 - 2ab\cos\gamma}$ and finally found the required proof.

Example:



ABC is the triangle having sides a, b, c respectively.

A having Angle α. , components of A(b, 0)

B having Angle β. , Components of B $(a \times \cos\gamma, a \times \sin\gamma)$

and C having Angle γ. , Components of C(0,0).

let consider $\overline{BD}$ is $\perp$ (perpendicular) to $\overline{DC}$, then the components of B would be $(a\cos\gamma, a\sin\gamma)$ hence $\overline{BC} = a$.

To find the $c = \overline{AB}$, By distance formula; $\overline{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$(AB)^2 = \left[\sqrt{(a\cos\gamma - b)^2 + (a\sin\gamma - 0)^2}\right]^2$

$c^2 = a^2\cos^2\gamma - 2ab\cos\gamma + b^2 + a^2\sin^2\gamma$

$c^2 = a^2(\cos^2\gamma + \sin^2\gamma) + b^2 - 2ab\cos\gamma$

$\because \cos^2\gamma + \sin^2\gamma = 1$. Similarly; $a^2 = b^2 + c^2 - 2bc\cos\alpha$

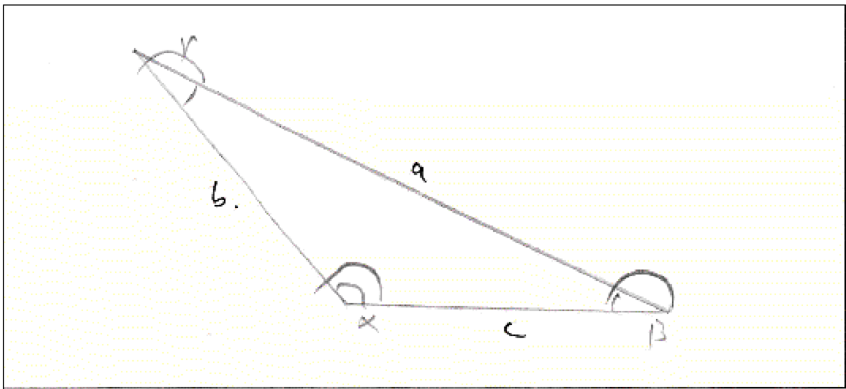
$c^2 = a^2 + b^2 - 2ab\cos\gamma$ HP. $b^2 = a^2 + c^2 - 2ac\cos\beta$.

Weaker responses reflected that the candidates failed to construct the correct diagram of the triangle *ABC* due to the fact that in the textbook, Point *A* is at origin but in the given proof, they were required to have point *C* at origin. Therefore, candidates failed to write the coordinates of the points correctly and consequently failed to prove the required results.

In few other weak responses, it was noted that candidates made mistakes in writing and in the application of distance formula and wrote $|AB| = \sqrt{(y_2 - x_1)^2 + (x_2 - y_1)^2}$ or $|AB| = \sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$, $|AB| = \sqrt{(y_2 - x_1)^2 + (x_2 - y_1)^2}$ or $|AB| = (x_2 - x_1) + (y_2 - y_1)$.

In few other responses, candidates directly wrote the law of cosine and wrote its different form. One such response is cited in the given example.

Example:



With the help of the ~~other~~ triangle the laws of cosines can be proved.

for the formula of $\cos\gamma =$

form $= c^2 = a^2 + b^2 - 2ab\cos\gamma.$

$\boxed{\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}.}$

for the formula of $\cos\beta =$ from

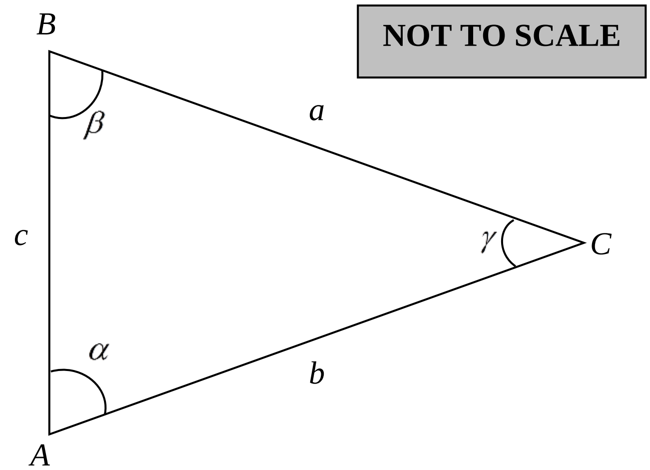
$b^2 = a^2 + c^2 - 2ac\cos\beta.$

$\boxed{\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}.}$

for the formula of $\cos\alpha = a^2 = b^2 + c^2 - 2bc\cos\alpha = \boxed{\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}.}$

Question 9b:

ii. In a triangle *ABC*, if the measure of $a = 6$ cm, $b = 7$ cm and $c = 3$ cm, then find the value of $\cos\frac{\alpha}{2}$. The semi perimeter *S* of the triangle *ABC* is equal to 8 cm.



Better responses exhibited that candidates selected the correct formula and substituted the values correctly in the formula to find the value of $\cos\frac{\alpha}{2}$

Example:

$$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}} = \sqrt{\frac{8(8-6)}{7\times 3}} = \sqrt{\frac{8(2)}{21}} = \sqrt{\frac{16}{21}} = \pm\frac{4}{\sqrt{21}}$$

$$\cos\frac{\alpha}{2} = \frac{4}{\sqrt{21}} \quad \because 0 \leq \frac{\alpha}{2} \leq \frac{\pi}{2}.$$

Weaker responses exhibited that candidates failed to write the formula correctly or made mistakes in substitution of values and hence failed to complete the question. Two such examples are cited below.

Example 1:

$$\frac{a+b+c}{2}=8 \qquad \cos\frac{\alpha}{2}=\frac{s(s-a)}{bc}$$

$$\frac{6+7+3}{2}=8 \qquad =\frac{8(8-6)}{7\times 3}$$

$$8=8 \qquad \cos\frac{\alpha}{2}=24\cdot 85$$

Example 2:

$$\cos\frac{\alpha}{2}=\sqrt{\frac{s(s-a)}{bc}}$$

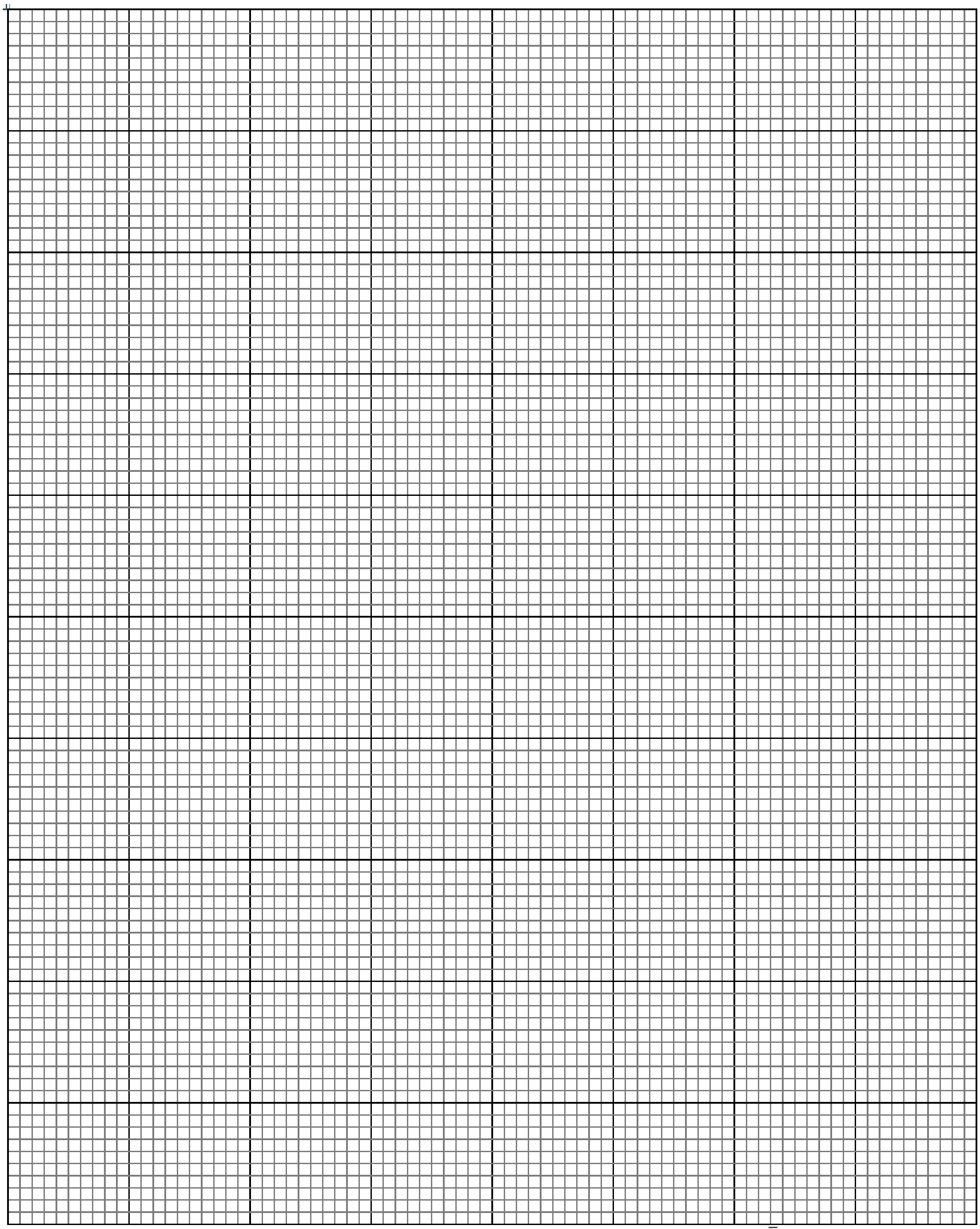
$$\cos\frac{\alpha}{2}$$

Question 10:

Question 10a:

Fill the following table and draw the graph of $\sec\theta$ on the given graph. Write the range of the $\sec\theta$.

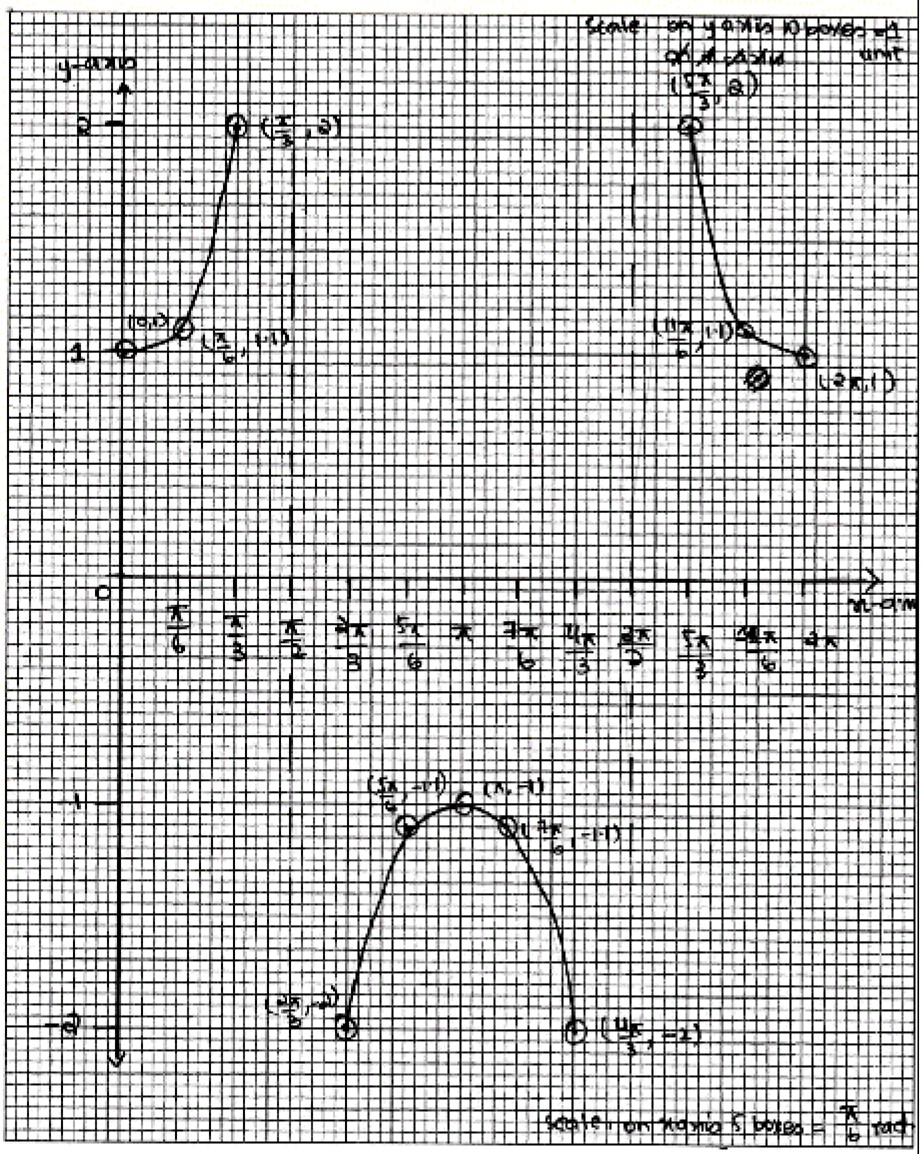
θ	$\sec\theta$
0	1
$\frac{\pi}{6}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	
$\frac{2\pi}{3}$	
$\frac{5\pi}{6}$	
π	
$\frac{7\pi}{6}$	
$\frac{4\pi}{3}$	
$\frac{3\pi}{2}$	
$\frac{5\pi}{3}$	
$\frac{11\pi}{6}$	
2π	



Better responses showed that candidates correctly calculated all values of $\sec\theta$ with the help of calculator and filled the given table. The candidates appropriately chose the scale on *x-axis* and *y-axis* and located the point on the given graph. Moreover, they skillfully marked the asymptotes of the graph. As a result, they were able to find the range of the $\sec\theta$.

Example 1:

θ	$\sec\theta$
0	~~0~~ 1
$\frac{\pi}{6}$	1.1
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	∞
$\frac{2\pi}{3}$	−2
$\frac{5\pi}{6}$	−1.1
π	−1
$\frac{7\pi}{6}$	−1.1
$\frac{4\pi}{3}$	−2
$\frac{3\pi}{2}$	$-\infty$
$\frac{5\pi}{3}$	2
$\frac{11\pi}{6}$	1.1
2π	1

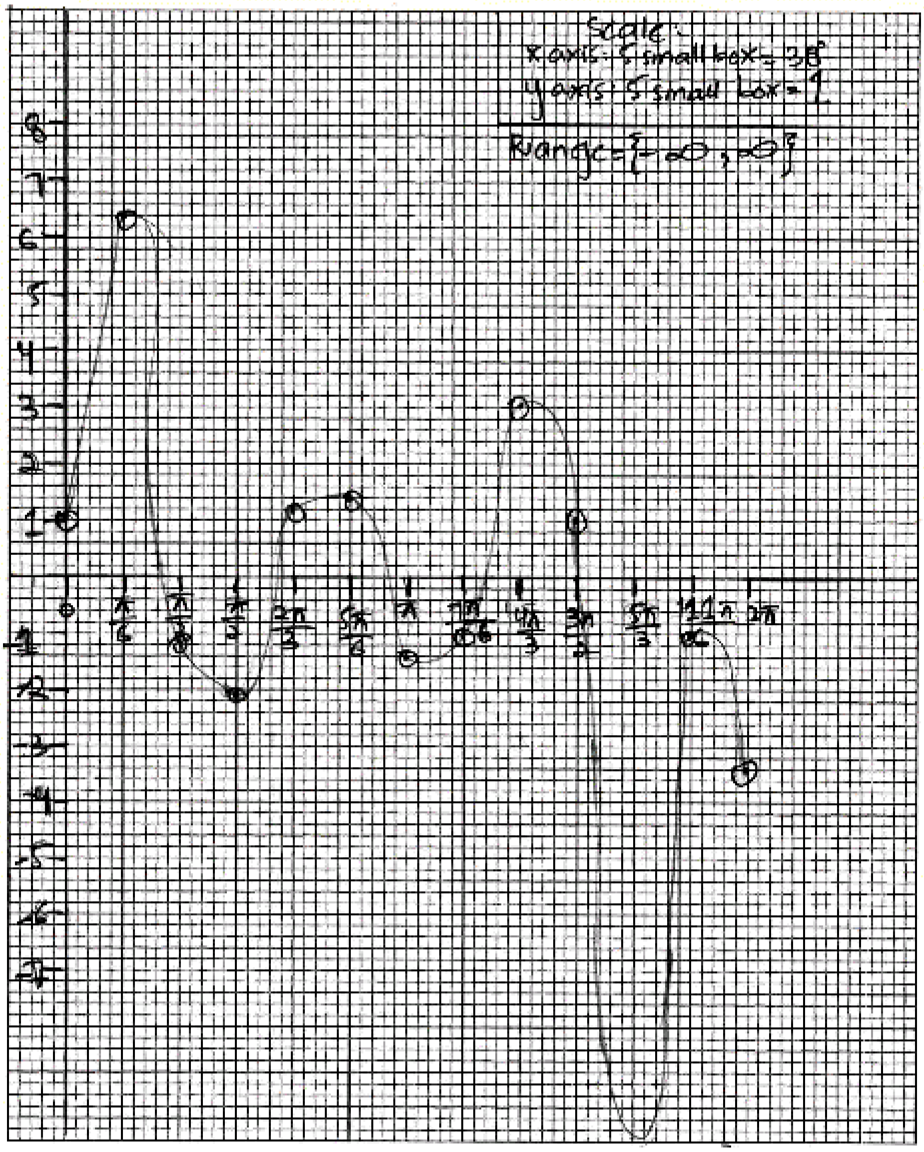


range of sec θ : $-1 \geq y \geq +1$

Weaker responses exhibited that candidates found the incorrect value of $\sec\theta$ for the given value of θ. Specifically, they failed to find the correct value of $\sec\theta$ at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. Similarly, they failed to locate values of $\sec\theta$ and θ on the given graph paper. In a few responses, it was noted that candidates failed to select appropriate scale on *x-axis* and *y-axis*. It was also evident from the weaker responses that candidates were clueless about the asymptotes of the given graph.

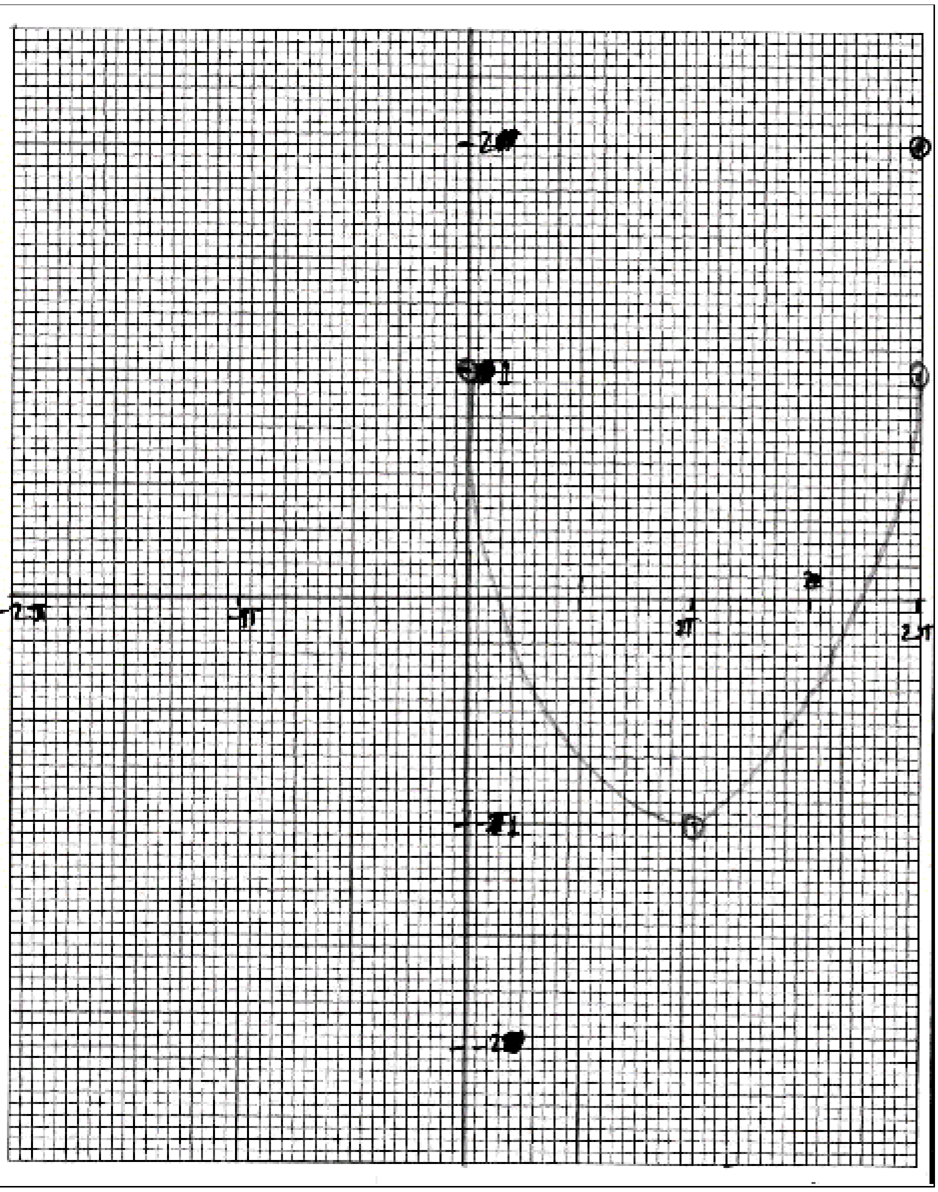
Example 1:

θ	$\sec\theta$
0	~~0~~ 1
$\frac{\pi}{6}$	6.48
$\frac{\pi}{3}$	-1.04
$\frac{\pi}{2}$	-2.28
$\frac{2\pi}{3}$	1.22
$\frac{5\pi}{6}$	1.41
π	-1.67
$\frac{7\pi}{6}$	-1.13
$\frac{4\pi}{3}$	3.06
$\frac{3\pi}{2}$	1.01
$\frac{5\pi}{3}$	-4.52
$\frac{11\pi}{6}$	-1.0
2π	-3.5



Example 2:

θ	$\sec\theta$
0	1
$\frac{\pi}{6}$	1.15
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-2
$\frac{5\pi}{6}$	2
π	-1
$\frac{7\pi}{6}$	2
$\frac{4\pi}{3}$	-2
$\frac{3\pi}{2}$	-1
$\frac{5\pi}{3}$	-2
$\frac{11\pi}{6}$	2
2π	1



Question 10b:

i. Solve $\cos x - \sqrt{3}\sin x = 0$

Better responses of part i, exhibited that candidates have good understanding of concepts of solution of trigonometric equations. They converted the given trigonometric equation as $\tan x = \frac{1}{\sqrt{3}}$ and found the values satisfying the equation $\tan x = \frac{1}{\sqrt{3}}$ i.e. $x = \frac{\pi}{6}$, $x = \frac{7\pi}{6}$ and finally wrote the general solution.

Example:

$\frac{\cos x}{\cos x} - \frac{\sqrt{3}\sin x}{\cos x} = 0$ (divide by $\cos x$)

$1 - \sqrt{3}\tan x = 0$

$\sqrt{3}\tan x = 1$

$\tan x = \frac{1}{\sqrt{3}}$

$x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$x = 30 \left(\frac{\pi}{6}\right)$

tan is positive in I and III quad with reference angle $\frac{\pi}{6}$

$x = \pi/6$

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$G.S = \left\{\frac{\pi}{6} + n\pi, \frac{7\pi}{6} + n\pi\right\}$

Weaker responses showed that candidates failed to apply the correct technique to solve the given equation. In a few cases, candidates were able to find the principal angle, however, they failed to find the other angle satisfying the given trigonometric equation and consequently failed to write the general solution of the equation.

Example 1:

$\cos^2 x - 3\sin^2 x = 0$

$\cos^2 x - 3 + 3\cos^2 x = 0$

$4\cos^2 x - 3 = 0$

$4\cos^2 x = 3$

$\cos^2 x = 3/4$

$\cos x = \sqrt{3}/2$

$x = \cos^{-1} \sqrt{3}/2$

$x = 30°$

$\therefore 3\sin^2 x + 3\cos^2 x = 3$

$\boxed{x = \pi/6}$ Ans

Example 2:

$\cos x = \sqrt{3}\sin x$

$\frac{\cos x}{\sin x} = \sqrt{3}$ $\therefore \frac{\cos x}{\sin x} = \cot x$

$\cot x = \sqrt{3}$

$x = \cot^{-1}\sqrt{3}$

$x = 60^\circ$

Question 10b: ii. Find the value(s) of x for the trigonometric equation $\cos 2x - \sin^2 x + 2 = 0$.

Better responses exhibited that candidates applied the formula of $\cos 2x = \cos^2 x - \sin^2 x$ and converted the given equation to $3 - 3\sin^2 x = 0$ or to $3\cos^2 x = 0$ and were able to find the values of x satisfying the given equations and then wrote the general solution of the given equation.

Example 1:

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos^2 x - \sin^2 x - \sin^2 x + 2 = 0$

$\cos^2 x - 2(\sin^2 x) + 2 = 0$

$\cos^2 x - 2(1 - \cos^2 x) + 2 = 0$

$\cos^2 x + 2\cos^2 x \cancel{-2} \cancel{+2} = 0$

$3\cos^2 x = 0$

$\cos^2 x = 0$

$\cos x = 0$ Solution set $\{\frac{\pi}{2} + 2n\pi\} \cup \{\frac{3\pi}{2} + 2n\pi\}$

$x = \cos^{-1} 0$ Answer ↗

$x = \pi/2, 3\pi/2$

Example 2:

$$\cancel{(\cos^2 x - \sin^2 x) - \sin^2 x + 2 = 0}$$
$$\cancel{\cos^2 x - 2\sin^2 x + 2 = 0}$$
$$(1 - 2\sin^2 x) - \sin^2 x + 2 = 0$$
$$1 - 2\sin^2 x - \sin^2 x + 2 = 0$$
$$-3\sin^2 x + 3 = 0$$
$$\sin^2 x - 1 = 0$$
$$\sin^2 x = 1$$
$$\sin x = \pm 1$$
$$\sin x = 1 \quad , \quad \sin x = -1$$
$$x = \frac{\pi}{2} \quad , \quad x = -\frac{\pi}{2} \qquad \therefore x = \left\{\frac{\pi}{2}, -\frac{\pi}{2}\right\}.$$

Weaker responses exhibited that candidates failed to apply the formula of $\cos 2x = \cos^2 x - \sin^2 x$ and then made different type of mistakes, e.g. $1 - \cos^2 x + \cos 2x + 2 = 0 \Rightarrow \cos x = 0$ and $\cos x + 2 = 0$ and, therefore, failed to solve the required equation. Other mistakes have been cited in the given examples.

Example 1:

$$\sin^2 x + \cos 2x + 2 = 0.$$
$$1 - \cos^2 x + \cos 2x + 2 = 0$$
$$(\cos x(\cos x + 2)) + 3 = 0$$
$$\cos x = 0 \qquad \cos x + 5 = 0.$$
$$x = \{-5\} \cos^{-1}$$
$$\cos^2 2x - \sin^2 2x \cdot \sin^2 x + 2 = 0.$$
$$\sin^2 x(\sin 2x - 1) + \cos^2 2x = 0.$$
$$(\sin^2 x)(\cos 2x) + \cos^2 2x = 0.$$

Example 2:

$\cos^2 x - \sin^2 x - \sin^2 x + 2 = 0$

$\cos^2 x - 2\sin^2 x + 2 = 0$

$\cos^2 x$ Divide by $\cos^2 x$

$\frac{\cos^2 x}{\cos^2 x} - \frac{2\sin^2 x}{\cos^2 x} + \frac{2}{\cos^2 x} = 0$

$1 - 2\tan^2 x + \frac{2}{\cos^2 x} = 0$

$\frac{1 - 2\tan x}{1 - \tan^2 x} + \frac{2}{\cos^2 x}$ $\quad \because 2\tan = \frac{1 - 2\tan x}{1 - \tan^2 x}$

$\frac{1 - \tan^2 x - 2\tan x}{1 - \tan^2 x} + \frac{2}{\cos^2 x}$

$\tan^2 x - 2\tan + 1$