**Aga Khan University Examination Board**

**Notes from E-Marking Centre on SSC-II General Mathematics Examination May 2017**

## Introduction

This document has been produced for the teachers and candidates of Secondary School Certificate (SSC-II) General Mathematics. It contains comments on candidates' responses to the 2017 SSC-II Examination indicating the quality of the responses and highlighting their relative strengths and weaknesses.

## E- Marking Notes

This includes overall comments on candidates' performance on every question and *some* specific examples of candidates' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

## General Observations:

Generally it is noted that weaker candidates are not well-versed with the hierarchy of arithmetical, algebraic operations, appropriate formulae and its application. This is generally obstructing their performance in overall paper of General mathematics.

**Detailed Comments:**

**Constructed Response Questions (CRQs)**

**Question 1ai:**

Simplify the given rational expression to its lowest term.

$\frac{4x^2-4x+1}{4x^2-1}$.

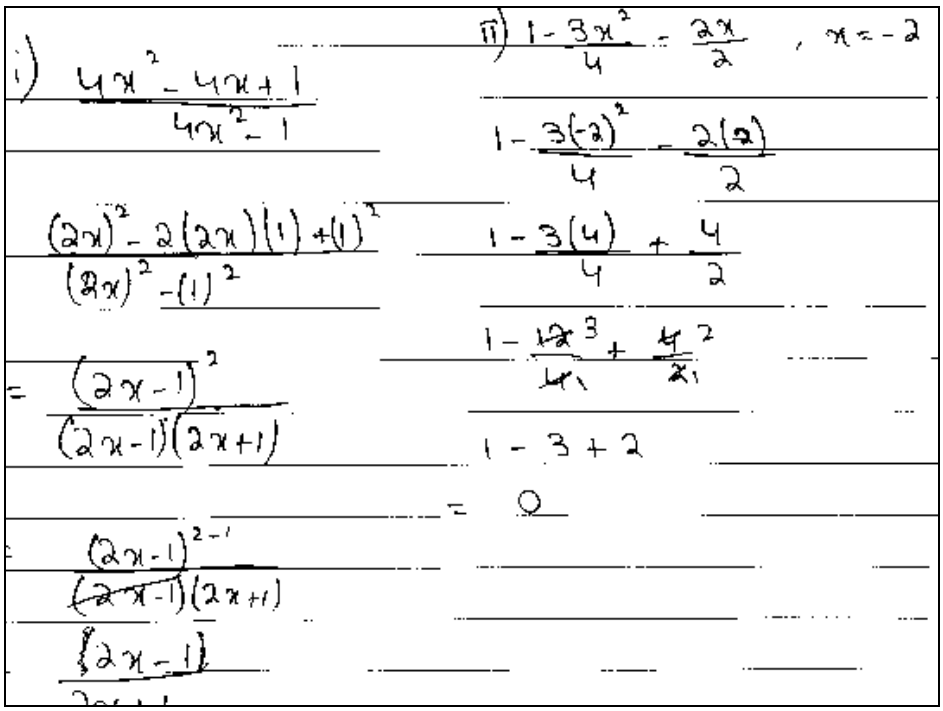
**Question 1aii:**

Find the value of $1-\frac{3x^2}{4}-\frac{2x}{2}$ at $x=-2$.

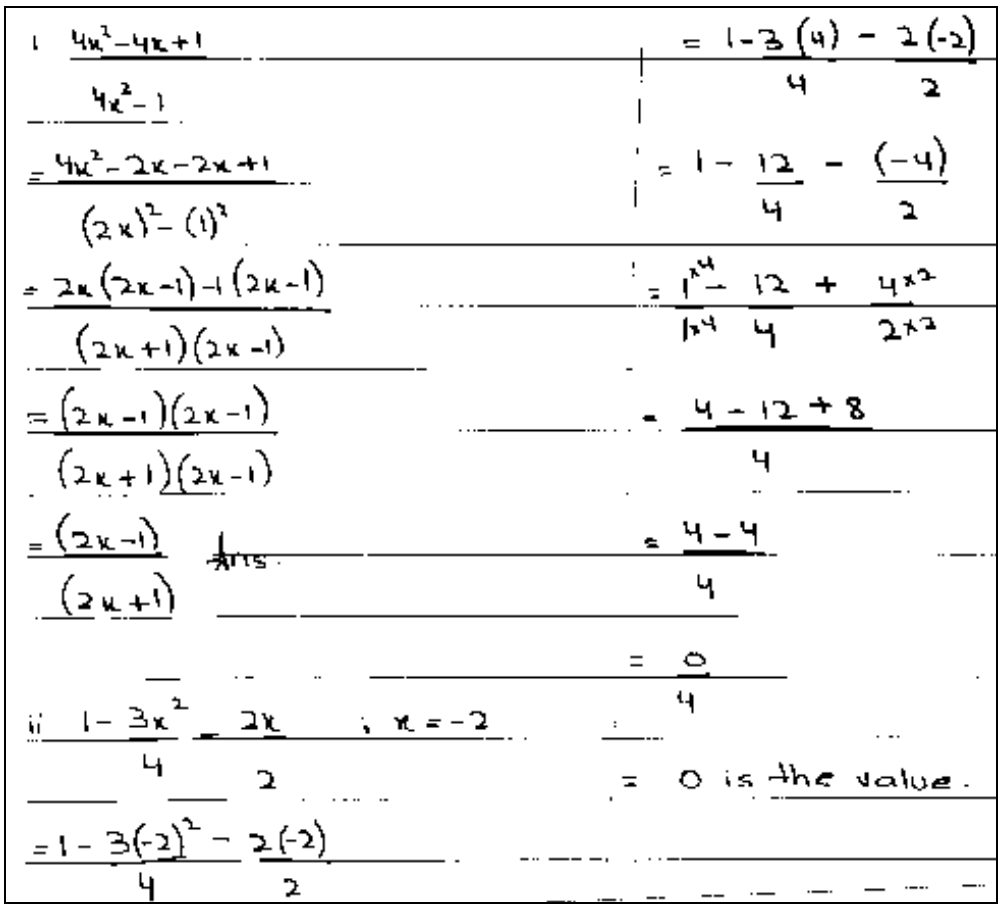
*Better responses* exhibited that candidates were able to apply the formulae

$x^2-y^2=(x+y)(x-y)$ and $(x-y)^2=x^2-2xy+y^2$ correctly to simplify the given algebraic expression. The candidates were also clear about the concept of substituting the value of variable in the expression to find the value of the expression.

**Example 1:**



**Example 2:**




*Weaker responses* indicated that candidates have confusions in writing and applying formula. Some mistakes are cited below.

$(x+y)^2 = x^2 + y^2$

$(x-y)^2 = x^2 - y^2$

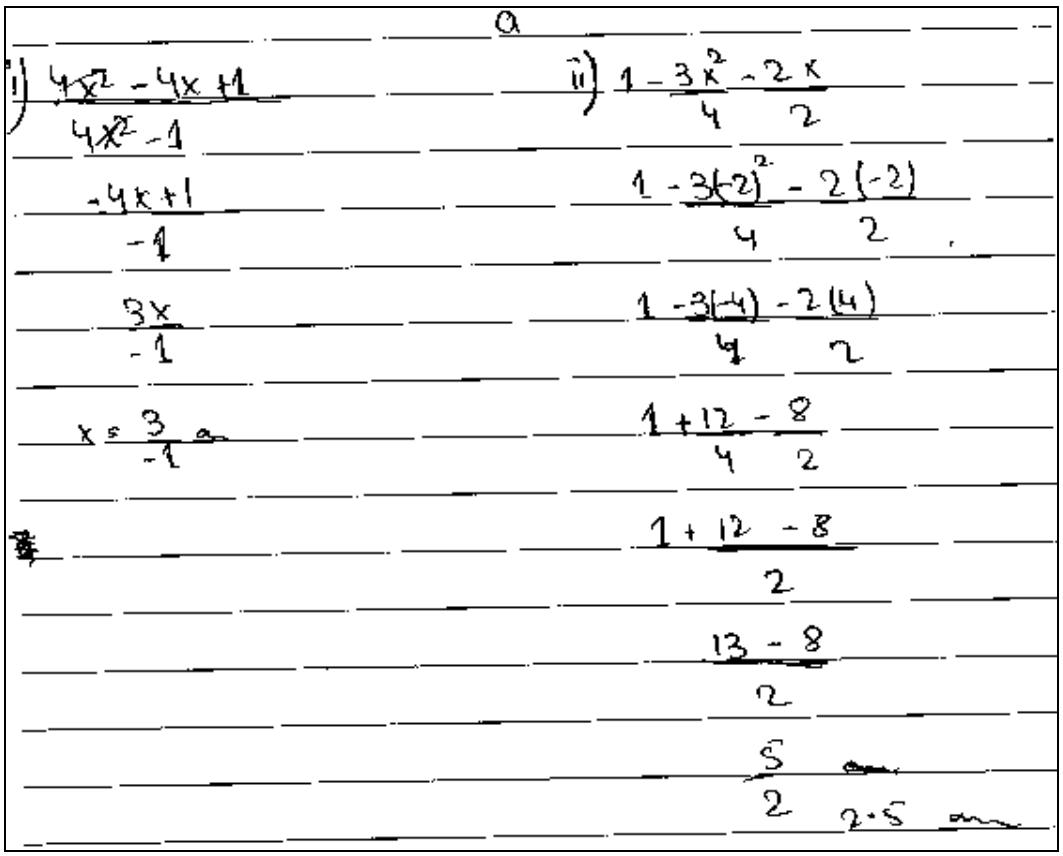
$\dfrac{4x^2-4x+1}{4x^2-1} = -4x$

In many such responses, it was noted that candidates started cancelling terms in numerator and denominator incorrectly as shown in the given examples.

In **part 1aii,** it was noted that to find the value of the given expressions, either candidates failed to handle the multiplication of signs (− and +) or made mistakes in taking the L.C.M. although it was not needed. In few other responses, it was noted that candidates substituted the value of the variable but failed to remove the variable.

One example of mishandling of sign is $\dfrac{4-12+8}{4} = \dfrac{4-20}{4}$

**Example 1:**



a

i) $\frac{4x^2 - 4x + 1}{4x^2 - 1}$

$\frac{-4x + 1}{-1}$

$\frac{3x}{-1}$

$x = \frac{3}{-1}$ a

ii) $\frac{1 - 3x^2 - 2x}{4 \quad 2}$

$\frac{1 - 3(-2)^2 - 2(-2)}{4 \quad 2}$

$\frac{1 - 3(-4) - 2(4)}{4 \quad 2}$

$\frac{1 + 12 - 8}{4 \quad 2}$

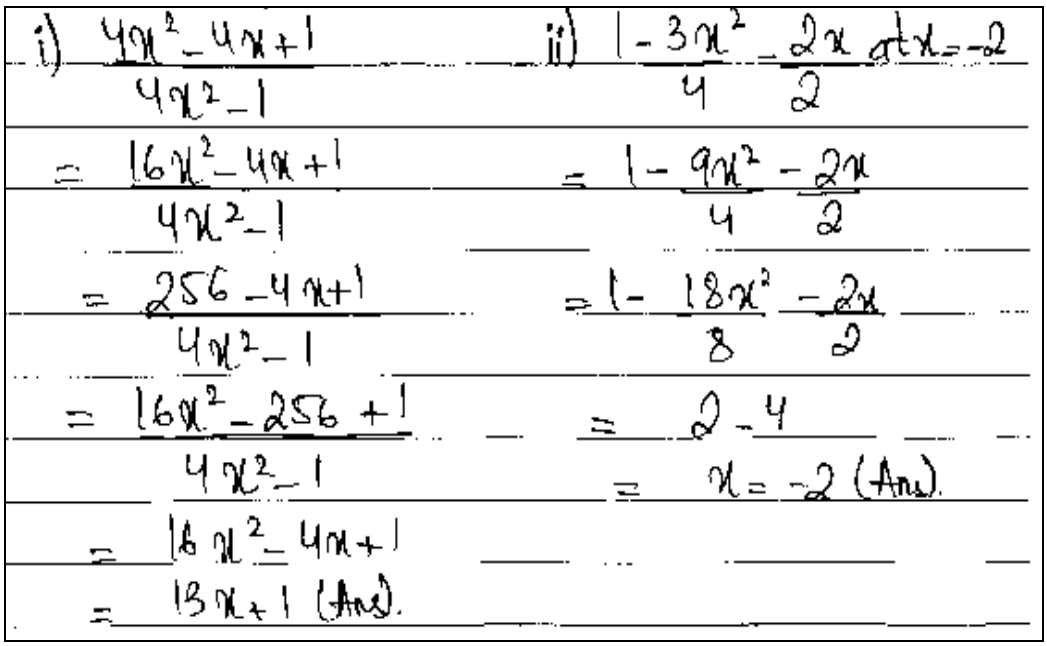
$\frac{1 + 12 - 8}{2}$

$\frac{13 - 8}{2}$

$\frac{5}{2}$ a

2.5 a

**Example 2:**



i) $\frac{4x^2 - 4x + 1}{4x^2 - 1}$

$= \frac{16x^2 - 4x + 1}{4x^2 - 1}$

$= \frac{256 - 4x + 1}{4x^2 - 1}$

$= \frac{16x^2 - 256 + 1}{4x^2 - 1}$

$= \frac{16x^2 - 4x + 1}{}$

$= 13x + 1$ (Ans).

ii) $\frac{1 - 3x^2 - 2x}{4 \quad 2}$ at $x = -2$

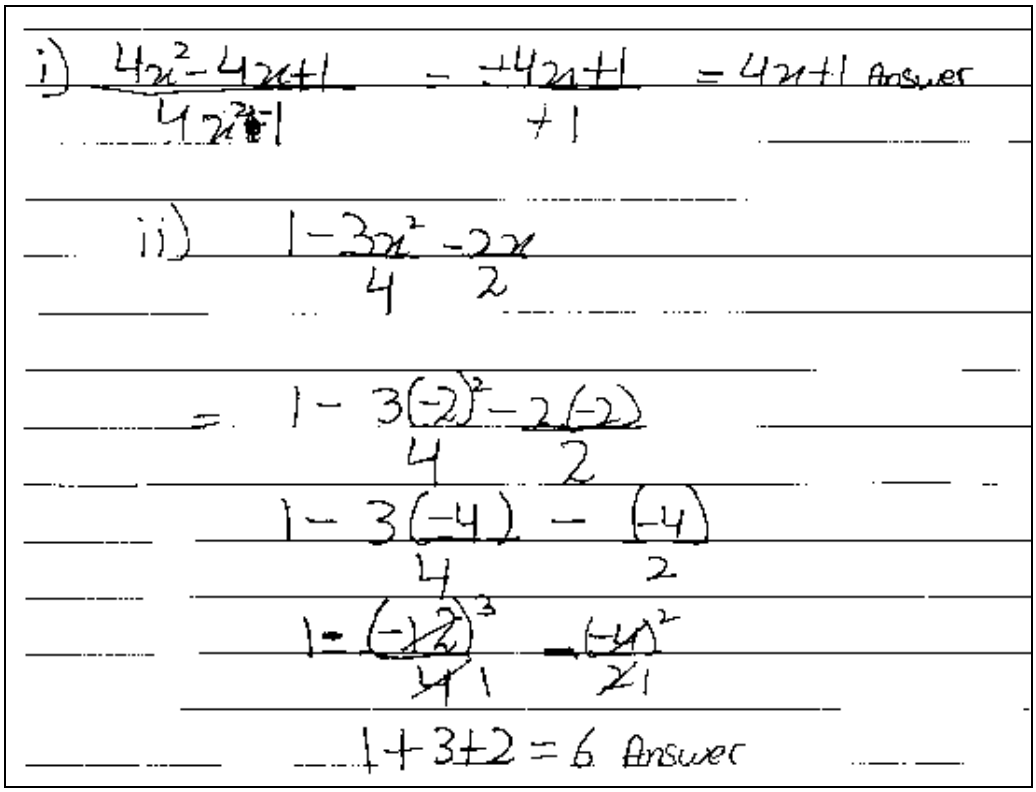
$= \frac{1 - 9x^2 - 2x}{4 \quad 2}$

$= \frac{1 - 18x^2 - 2x}{8 \quad 2}$

$= 2 - 4$

$= x = -2$ (Ans).

**Example 3:**


i) $\frac{4x^2-4x+1}{4x^2-1} = \frac{+4x+1}{+1} = 4x+1$ Answer
ii) $1-\frac{3x^2}{4}-\frac{2x}{2}$
$= 1-\frac{3(-2)^2}{4}-\frac{2(-2)}{2}$
$1-\frac{3(-4)}{4}-\frac{(-4)}{2}$
$1-\frac{(-12)^3}{4\ 1}-\frac{(-4)^2}{2\ 1}$
$1+3+2=6$ Answer

**Question 1b:**

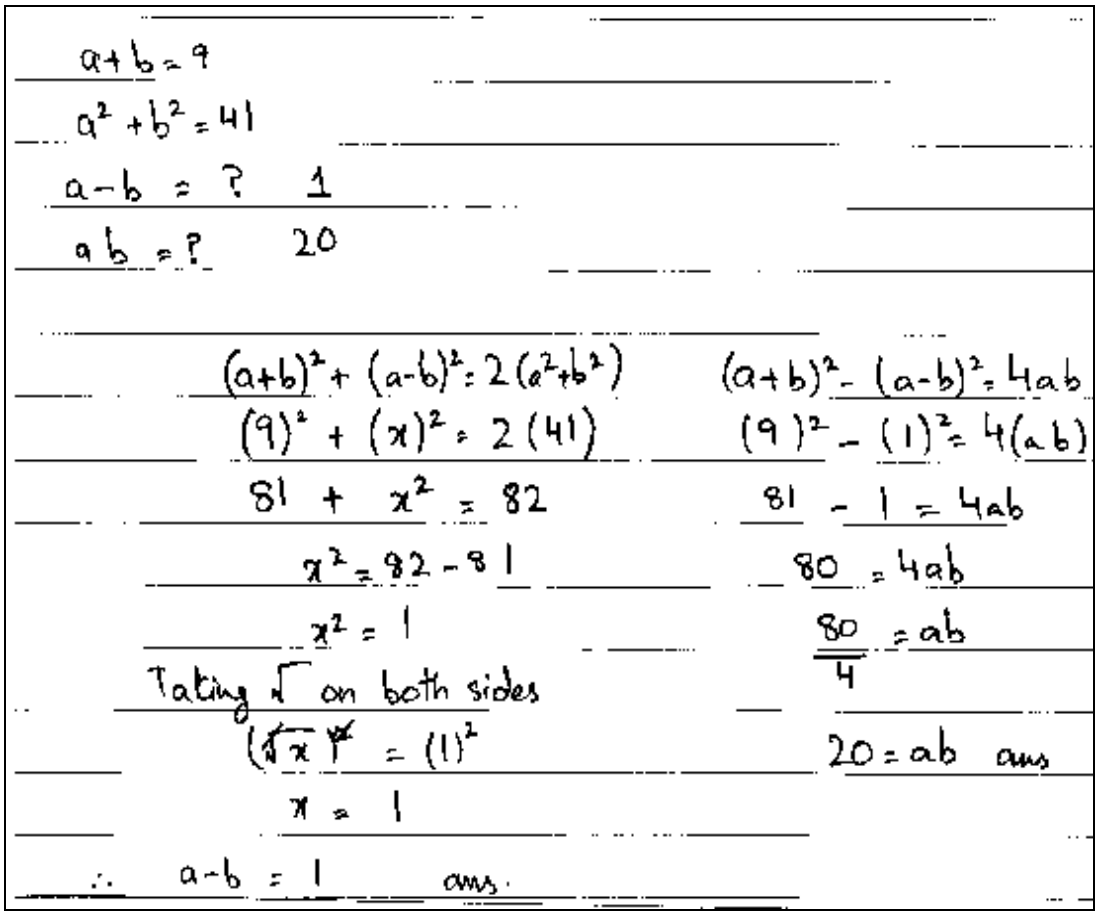
i. Given that $a+b=9$ and $a^2+b^2=41$, find the value of $a-b$ using algebraic method.

ii. Use values of part i. to find the value of $4ab$.

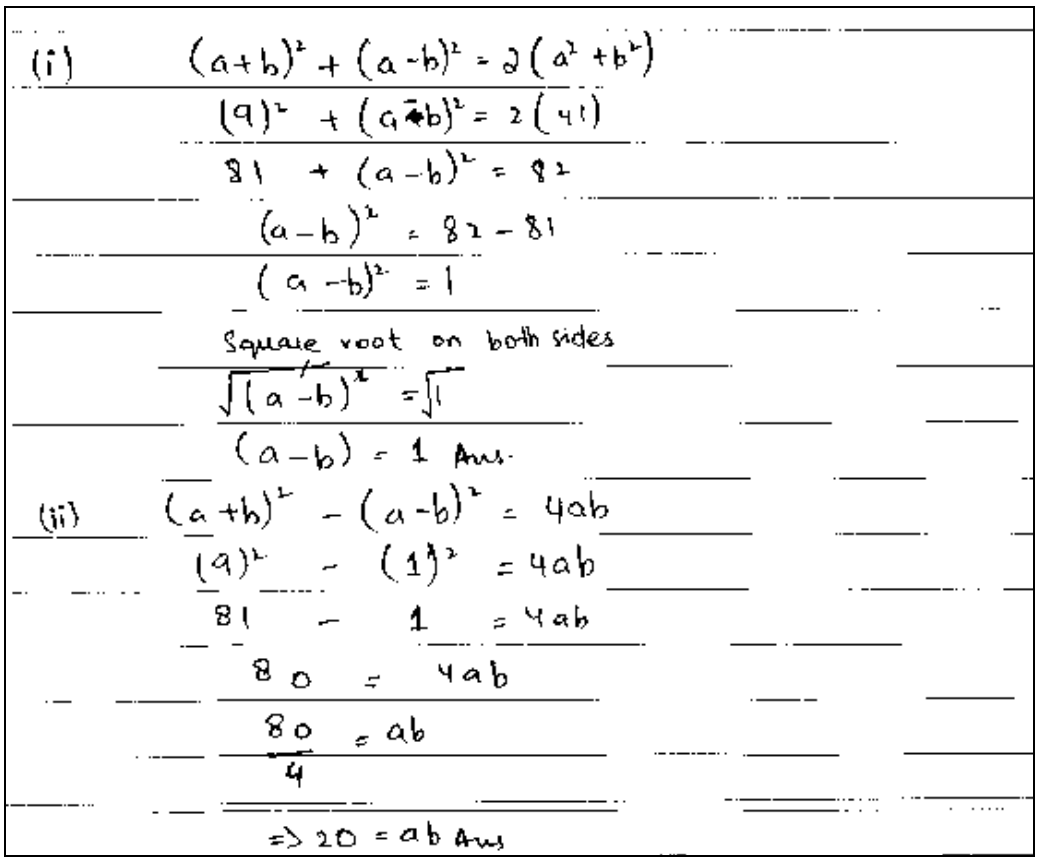
This part was opted by most of the candidates and was generally a well attempted question.

*Better responses* displayed that most of the candidates directly used the formula of $(a+b)^2+(a-b)^2=2(a^2+b^2)$ and after simplification got the value of $a-b$.

**Example 1:**



**Example 2:**




*Weaker responses* exhibited that candidates are not well acquainted with the formulae to be used in the given question and its application. Some frequent mistakes were:

$(a+b)=9$
$(a^2+b^2)=81$

Or

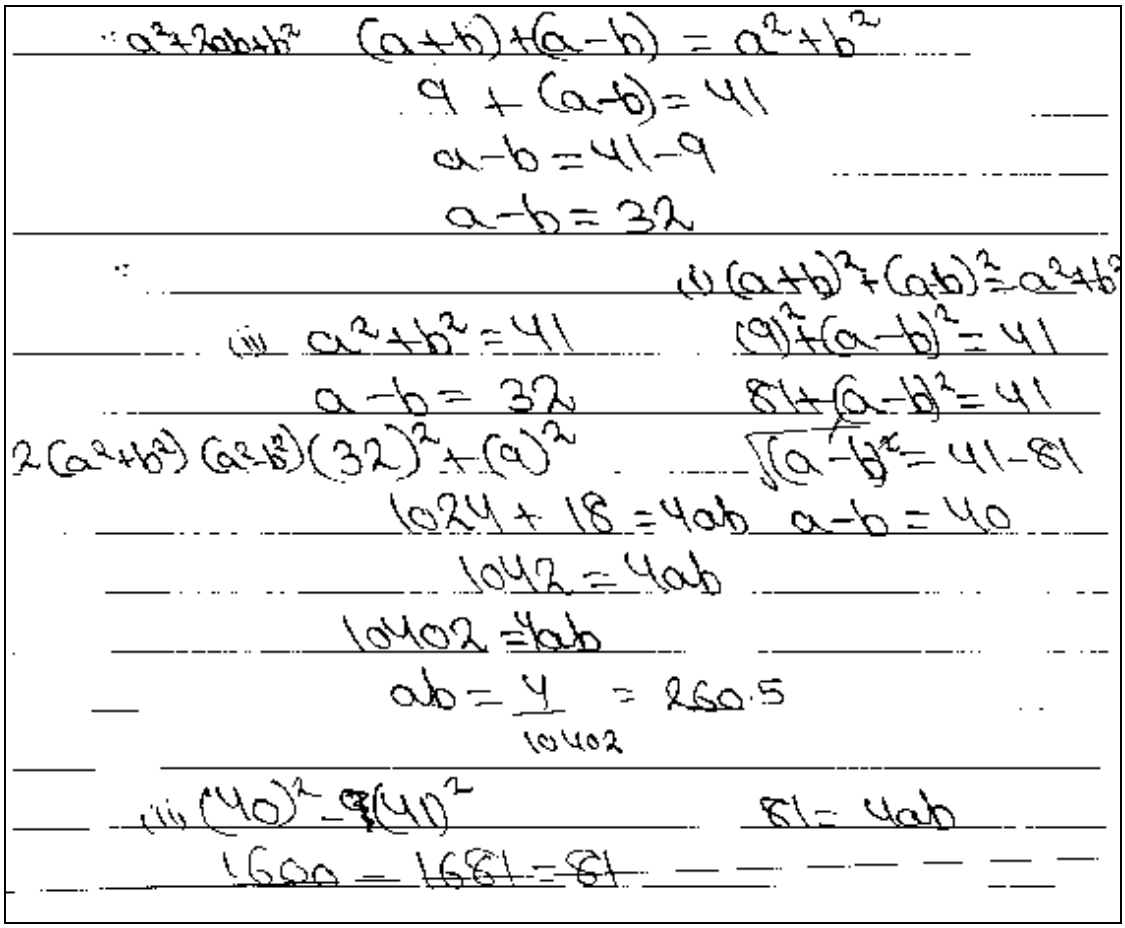
$(a+b)=5$
$(a^2+b^2)+(a^2-b^2)=2(a^2+b^2)$
$(a+b)+(a-b)=a^2+b^2$
$(a+b)(a-b)=4ab$
$a^2+b^2=(a-b)(a+b)$
$41=9(a-b)\Rightarrow a-b=32$

Further mistakes in writing the formula are clear in the given examples.

**Or**

Other candidates wrote the correct formulae but made mistakes in substitution of values or in simplification process. Even in better responses, some candidates missed ± sign and just wrote the positive value of the square root.

**Example 1:**



**Example 2:**

Solution:

Part i

Given: $a^2+b^2=41$

$a+b=9$

$a-b=?$

$a^2+b^2=(a+b)(a-b)$

$41=(9)(a-b)$

$41-9=a-b$

$32=a-b$

$a-b=32$ Ans

Part ii

$(a+b)(a-b)=4ab$

$(9)(32)=4ab$

$288=4ab$ Ans

$\frac{288}{4}=ab$

$\frac{288}{4}=ab$

$72=ab$.

**Example 3:**

(i) Given $a+b=9$ $a^2+b^2=41$ $a-b=?$

$a^2+b^2=(a+b)(a-b)$

$41=(9)(a-b)$

$41\div 9=a-b$

$a-b=41-9$

$a-b=32$

Verification: $(a+b)+(a-b)=a^2-b^2$

$9+32=41$

$41=41$

Hence proved.

(ii) $(a+b)(a-b)=4ab$

$9+32=4ab$

$41=4ab$

$ab=\frac{41}{4}$ $ab=10.25$

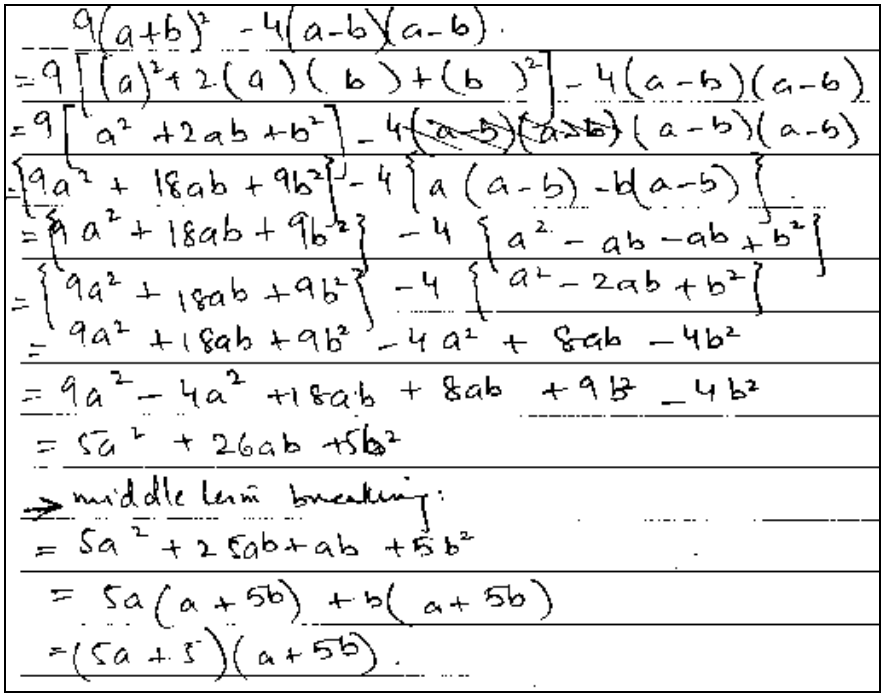
**Question 2a:**

Factorise completely the given expression $9(a+b)^2-4(a-b)(a-b)$.

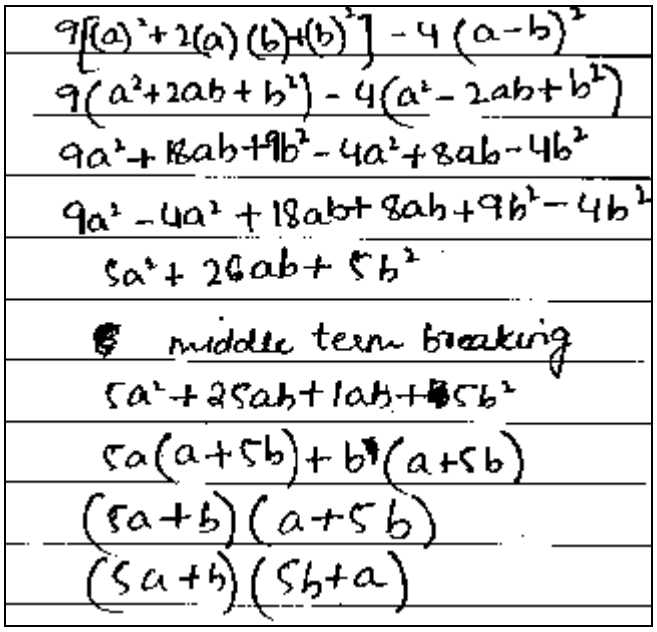
The candidates were offered choice between part **a** and part **b** of the question. Very few candidates opted for part **a** and generally it was not a well attempted question**.**

*Better responses* showed a clear understanding of factorisation. They solved the question systematically by application of appropriate formulae and were able to completely factorise the given expression. Alternatively, candidates expanded the given expression first, then simplified the expression and used breaking of middle term method to factorise the given expression.

**Example 1:**



**Example 2:**

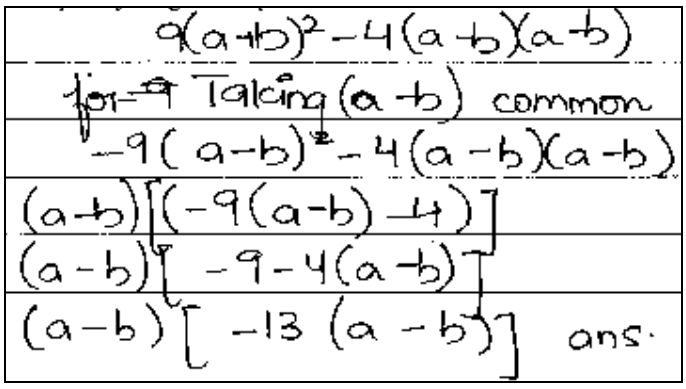



*Weaker responses* showed that the candidates were either not familiar with the formulae or failed to understand the factorisation process of the given question. Few common mistakes noted in the responses are cited in the examples to highlight the misconceptions of the candidates.

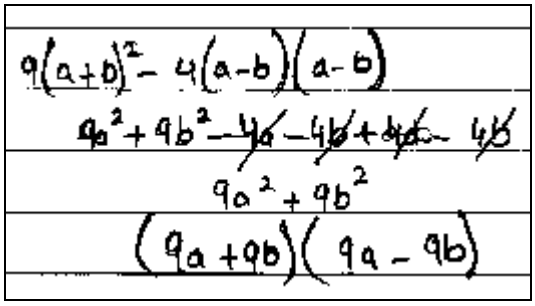
$(a+b)(a-b)=4ab$

$a^2+b^2=(a-b)(a+b)$

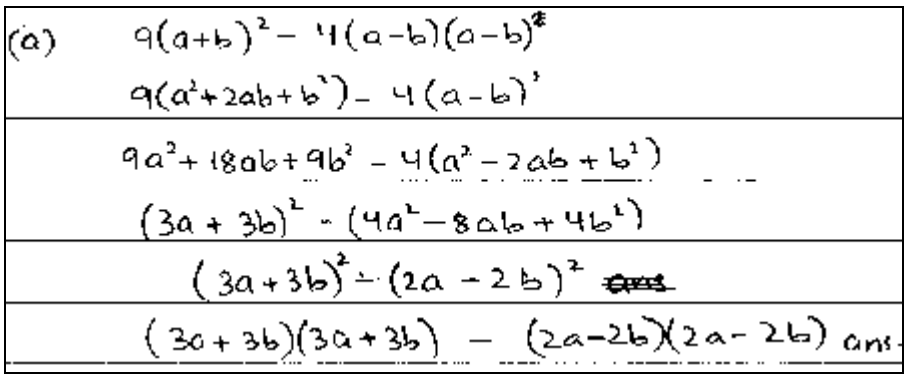
**Example 1:**



**Example 2:**




**Example 3:**



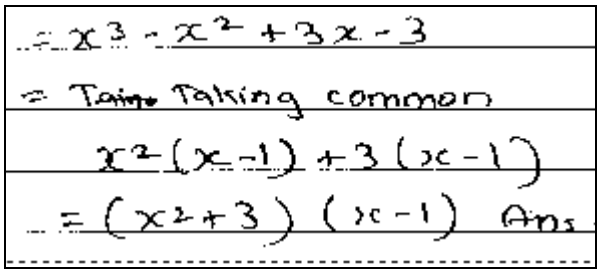

**Question 2b**:

Factorise completely the given expression $x^3 - x^2 + 3x - 3$.

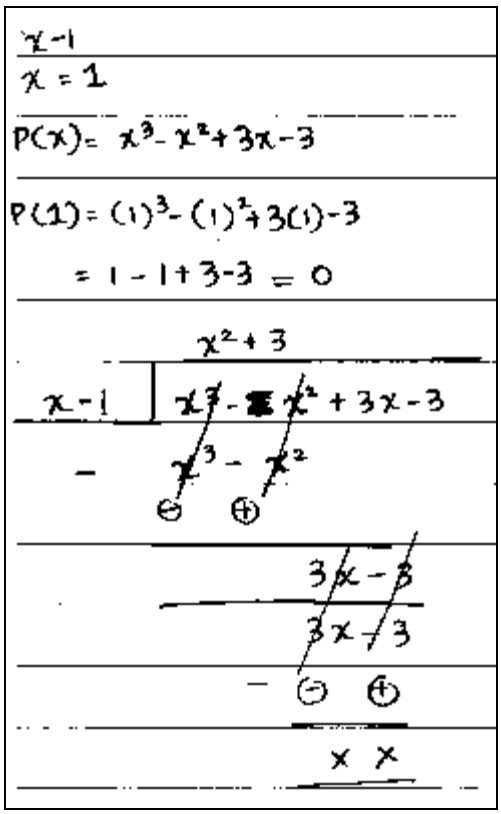
Generally this was a well attempted question.

*Better responses* displayed that the candidates factorised the given algebraic expression with correct technique.

**Example 1:**



**Example 2:**

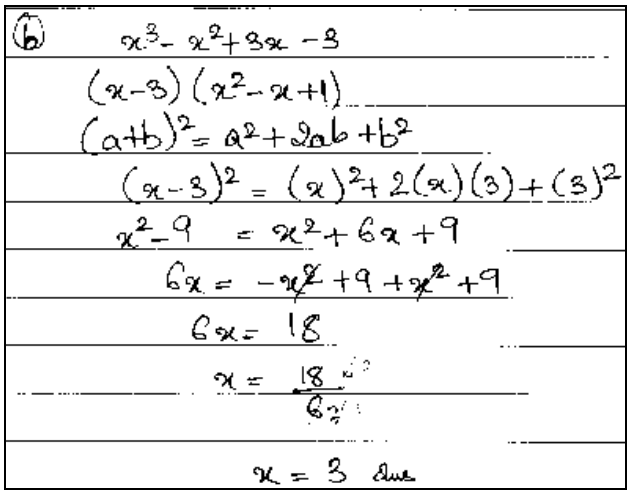



*Weaker responses* showed candidates' lack of understanding of basic algebraic operations, concept of factorisation and algebraic formulae. Some examples are cited below.

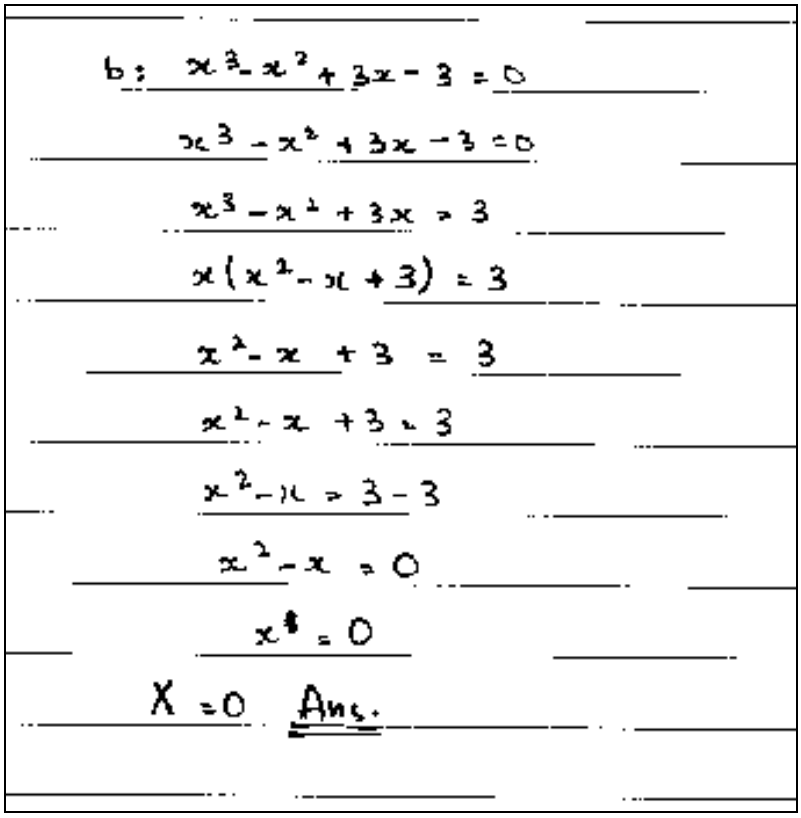
- $x^2 - 9 = x^2 + 6x + 9$
- $(x^3 - x^2 + 3x - 3) = (x-3)(x^2 - x + 1)$
- $(x^3 - x^2 + 3x - 3) = x(x^2 - x) + 3(x - 0)$

The following examples indicate some other misconceptions.

**Example 1:**



**Example 2:**

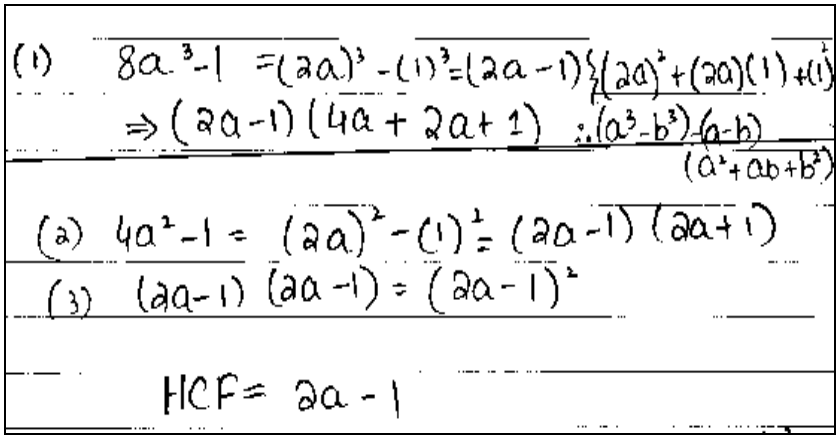



**Question 3a**:

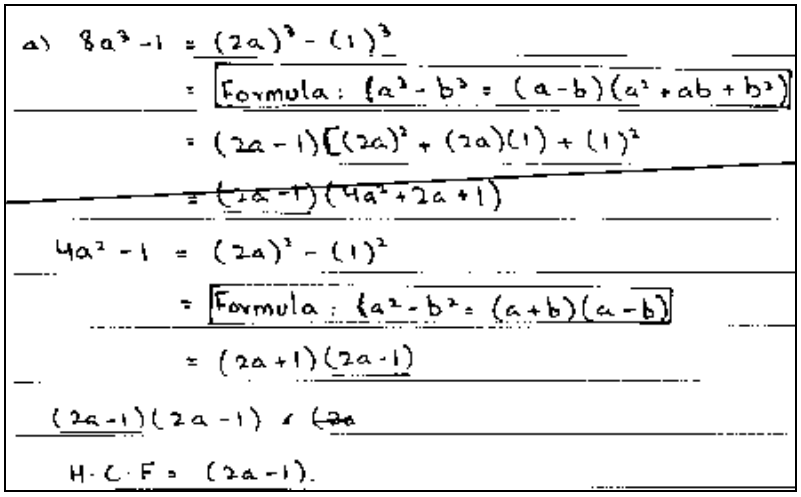
Find the highest common factor (H.C.F.) of $8a^3 - 1$, $4a^2 - 1$ and $(2a-1)(2a-1)$.

*In better responses,* candidates factorised the given expressions and hence were able to find the H.C.F. of the given expressions.

**Example 1:**

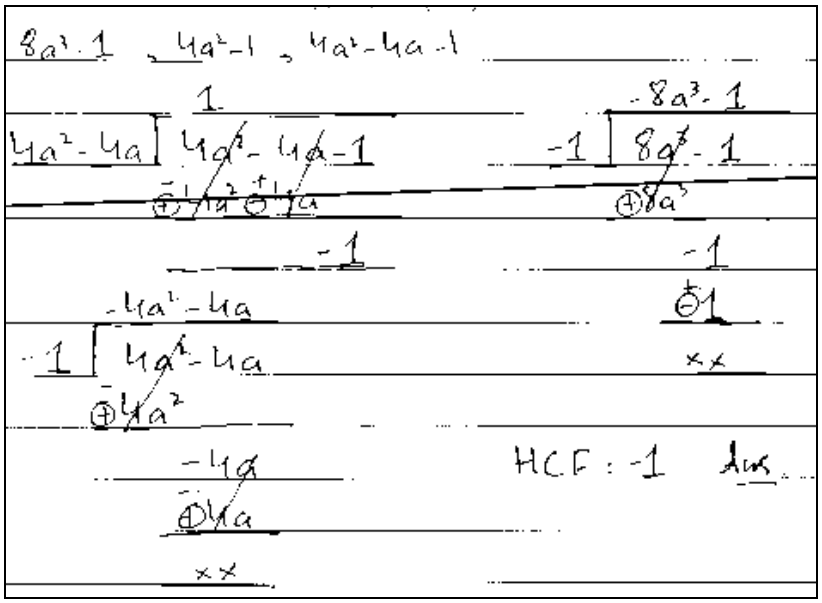


**Example 2:**

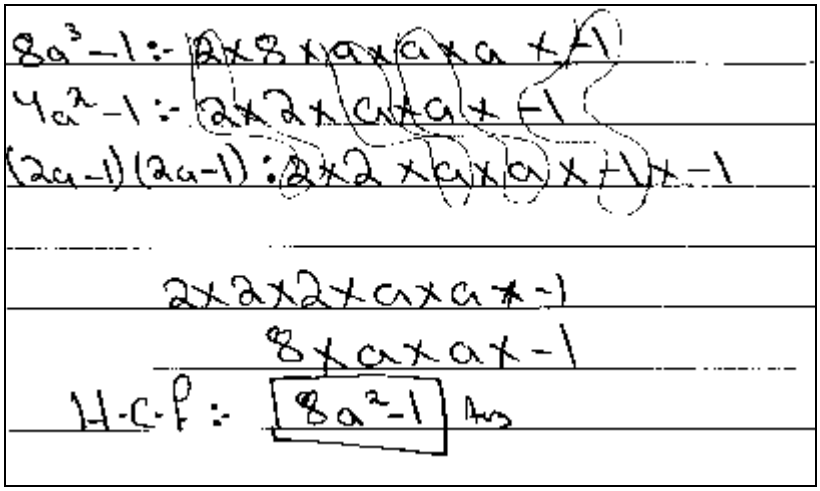



*Weaker responses* showed that candidates failed to factorise the given expression due to incorrect use of formulae. Some responses indicate that those candidates applied the division method were unable to apply it correctly. The incorrect factorisation cited in Example 2 was observed in many responses.

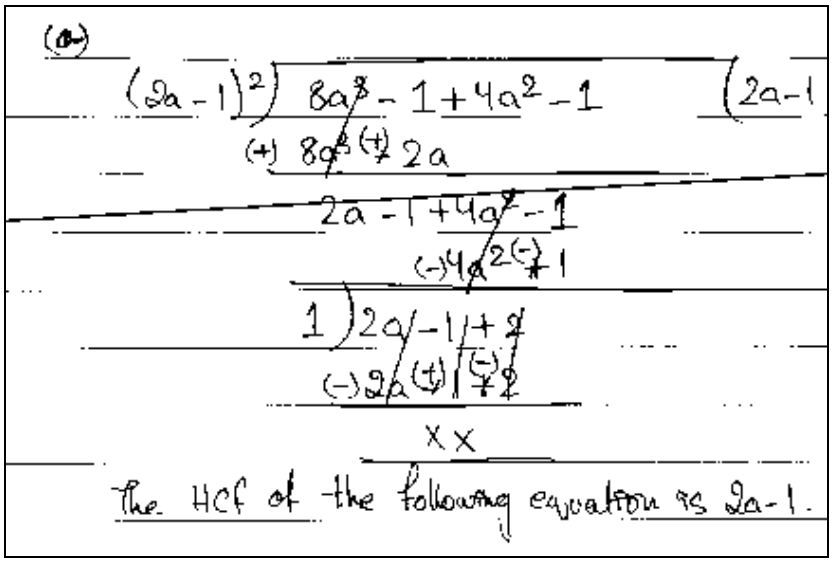
**Example 1:**



**Example 2:**




**Example 3:**



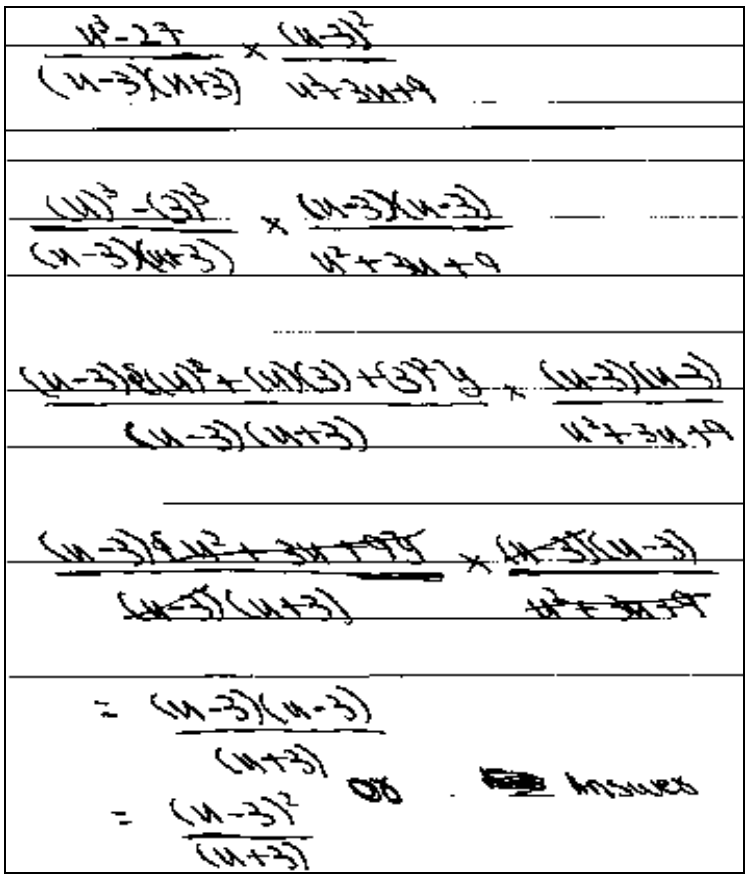
**Question 3b:**

Simplify the expression $\dfrac{x^3 - 27}{(x-3)(x+3)} \div \dfrac{x^2+3x+9}{(x-3)^2}$.

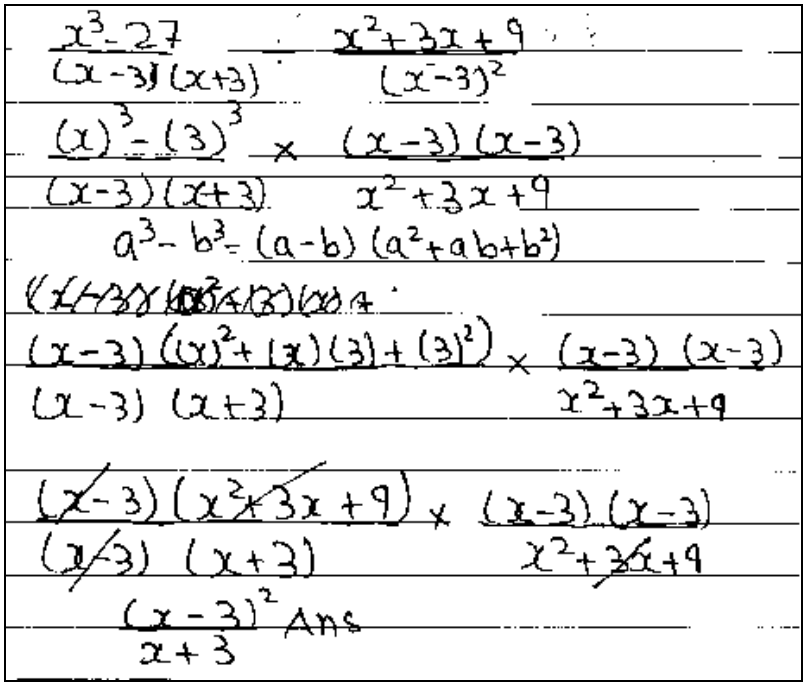
This question was attempted by fewer candidates and generally candidates were unable to perform well in it.

In *better responses,* candidates first converted division sign to multiplication sign cancelled the factor $(x-3)$ and applied formula of $x^3 - 3^3$ and through further cancellation, they were able to simplify the overall expression to get the required simplest form.

**Example 1:**

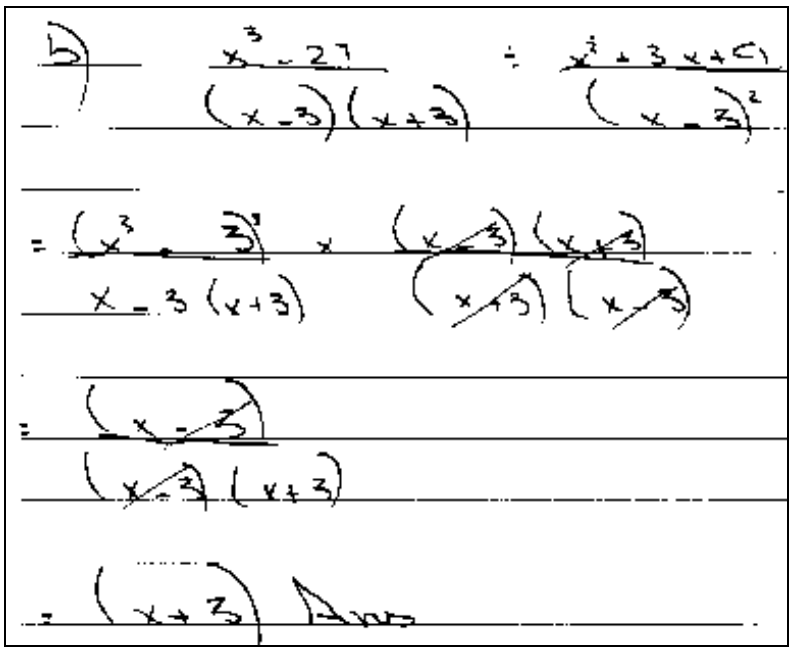


**Example 2:**

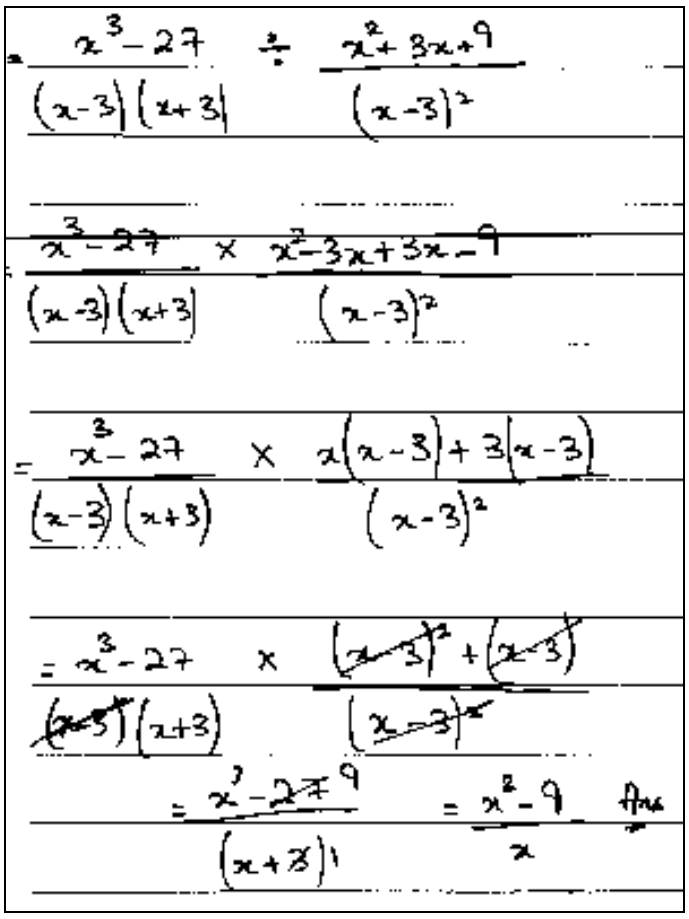



*Weaker responses* failed to apply formula of $x^3-3^3$ correctly and various mistakes were seen in cancellation of terms. Some responses reported that candidates, instead of simplifying, started expanding the given expression. To highlight such mistakes, three examples are cited below.

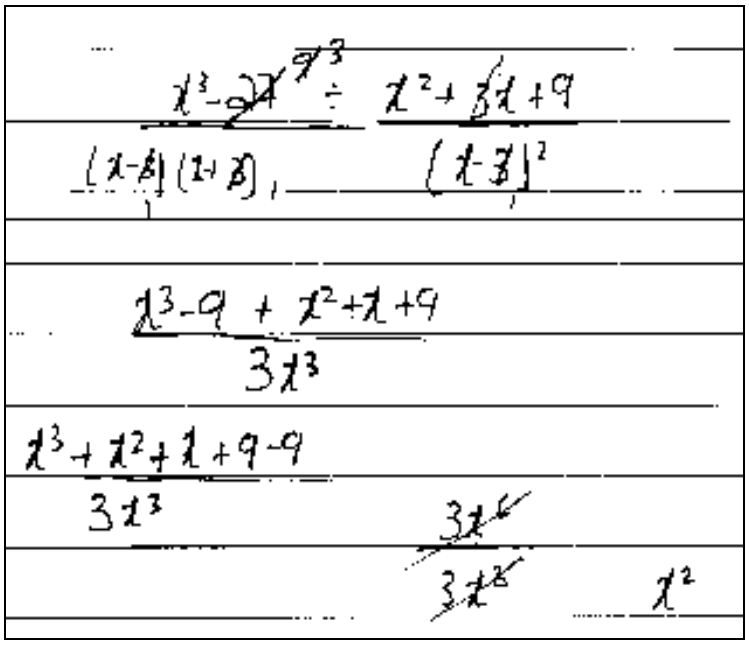
**Example 1:**



**Example 2:**




**Example 3:**



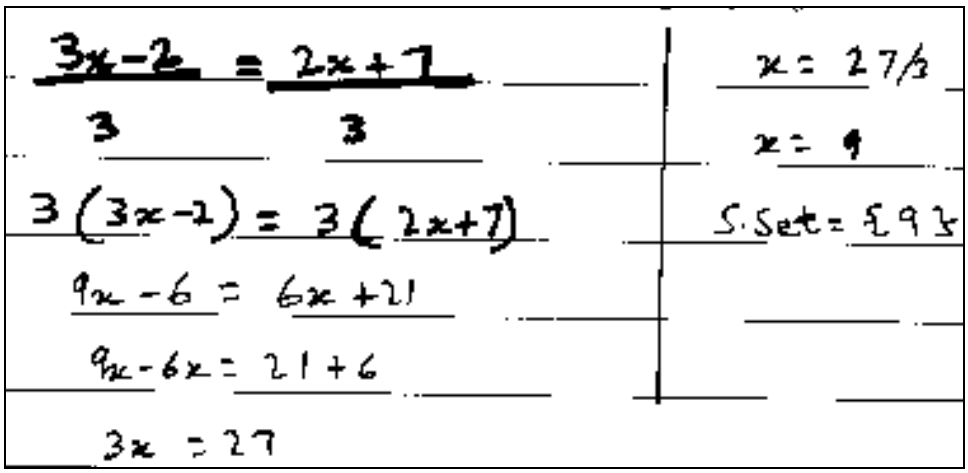
**Question 4:**

i. Find the solution set of the linear equation $x-\frac{2}{3}=\frac{2x}{3}+\frac{7}{3}$.

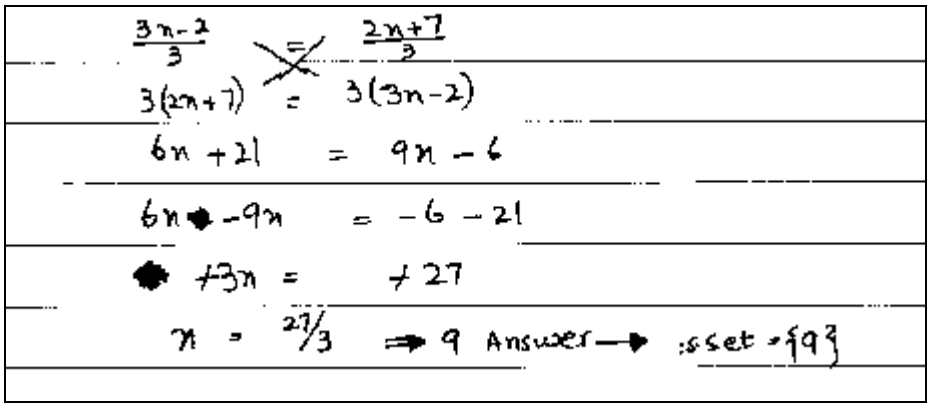
This was generally a well attempted question and candidates were able to find the value of *x.*

In *better responses,* many candidates either took L.C.M. and multiplied both the sides of the equation $x-\frac{2}{3}=\frac{2x}{3}+\frac{7}{3}$ by the L.C.M. to find the value of *x* or collected the term containing *x* on one side of the equation and constant terms on the other side of the equation. Through either of the methods they were able to successfully find the solution set of the equation.

**Example 1:**



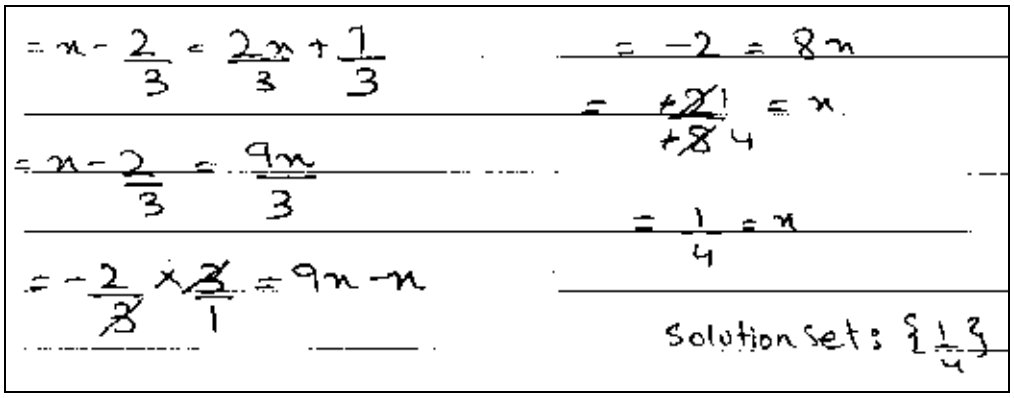

**Example 2:**



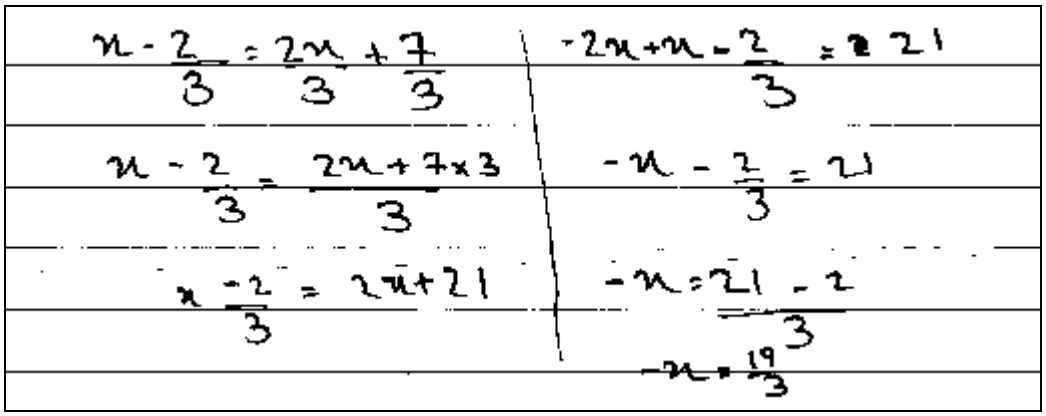
*Weaker responses* showed that candidates had various misconceptions about solution of linear equation; therefore, failed to find the values of *x*. Some mistakes noted in responses are listed below.

- $2x+7=9x$
- $2x+\frac{x}{3}=-7$
- $x+\frac{x}{3}=-7-2$
- $x-\frac{2}{3}=\frac{x-2}{3}$
  $2x+x=-21$

**Example 1:**




**Example 2:**



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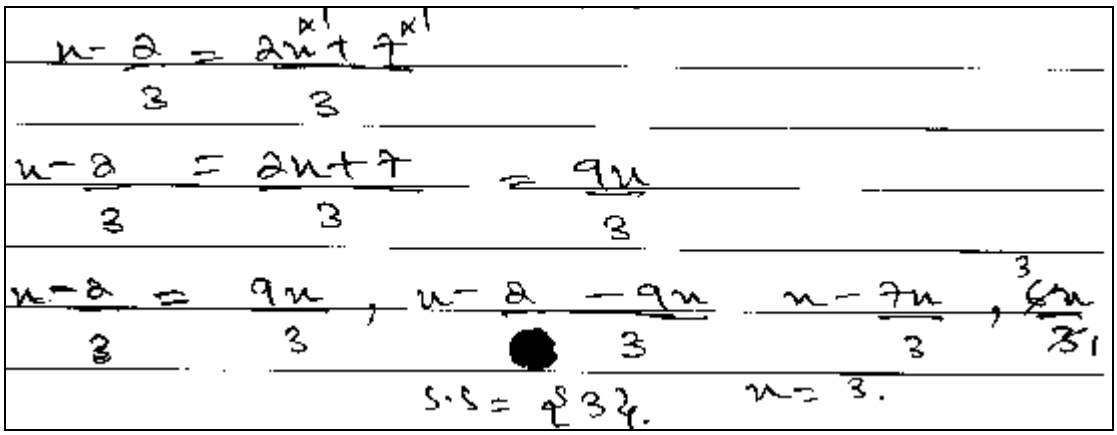
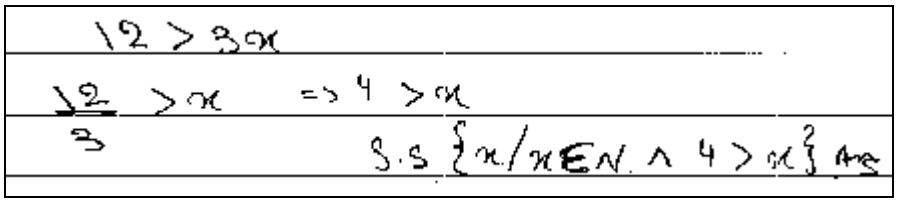



**Question 4:**

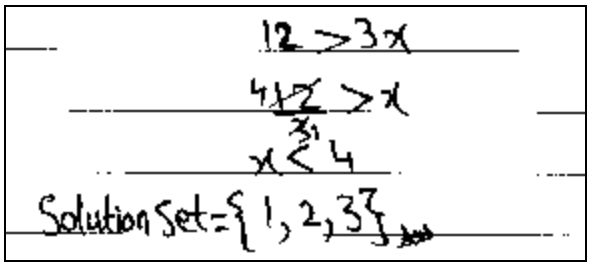
ii. Find the solution set of $12 > 3x$, where $x \in N$.

*Better responses* exhibited that candidates correctly applied basic division operation on the given inequality and consequently were able to find the solution set either in tabular form or in set builder form.

**Example 1:**




**Example 2:**



*Weaker responses* displayed different misconception in solving inequality and consequently failed to find the solution set of the given inequality.

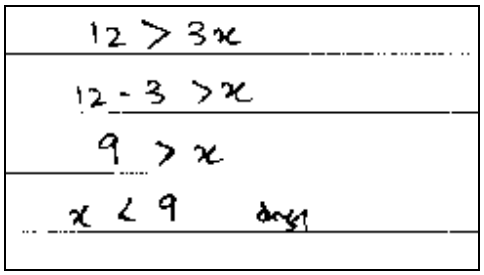
Few of them are as follows:

- $12 > 3x$
  $x = 12 - 3$
  $x = 9$

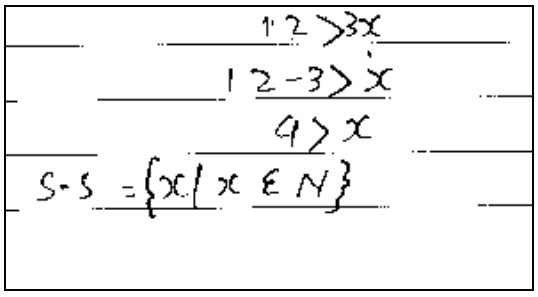
- $12 > 3x$
  $12 - 3 > x$
  $x < 9$

- $12 > 3x$
  $3x > 12$

**Example 1:**




**Example 2:**



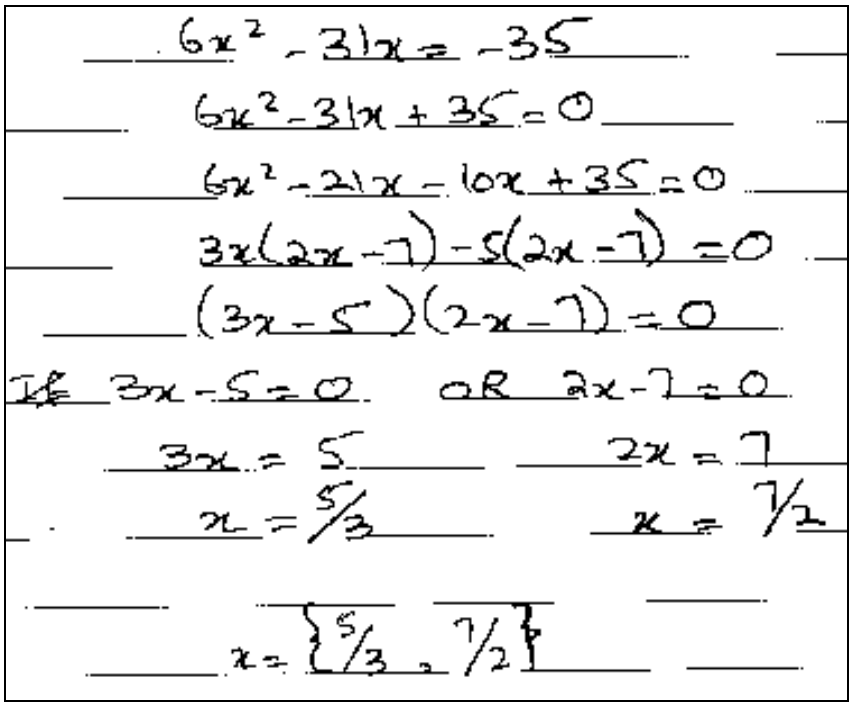
**Question 5a:**

Find the values of $x$ for $6x^2 - 31x = -35$

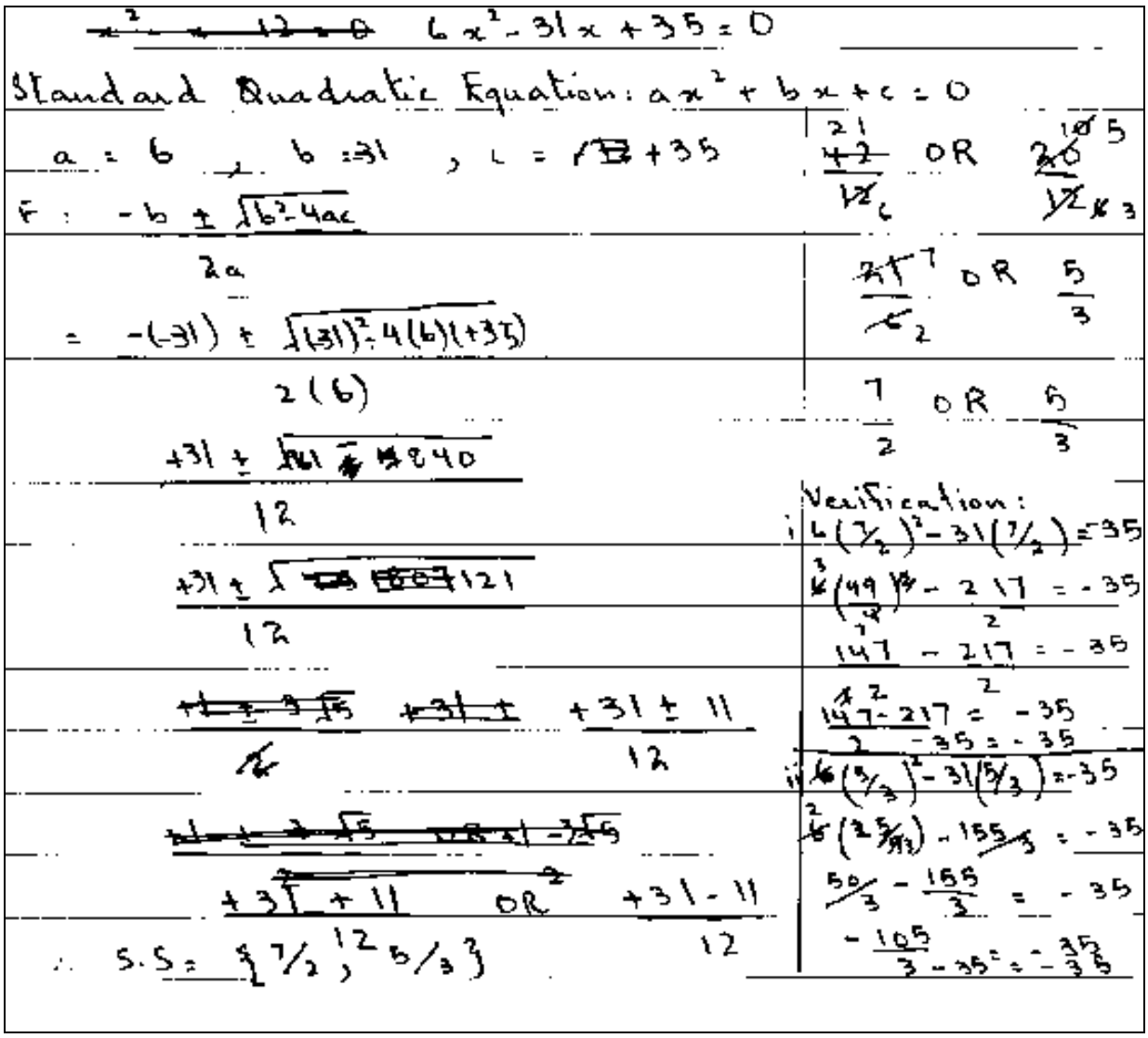
The question offered choice between part **a** and part **b**. Most of the candidates attempted part **a** and it was generally a well attempted question.

*Better responses* indicated that candidates had command over the solution of quadratic equation and applied quadratic formula or factorisation method to solve the question. In some responses, the method of completing square was also used to solve the given quadratic equation.

**Example 1:**



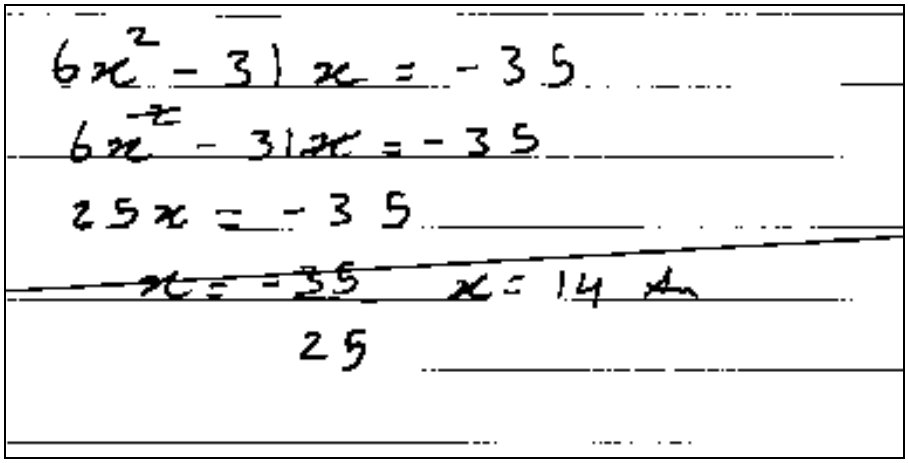
**Example 2:**


$6x^2 - 31x + 35 = 0$
Standard Quadratic Equation: $ax^2 + bx + c = 0$
$a = 6$, $b = -31$, $c = +35$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$= \frac{-(-31) \pm \sqrt{(-31)^2 - 4(6)(+35)}}{2(6)}$
$\frac{+31 \pm \sqrt{961 - 840}}{12}$
$\frac{+31 \pm \sqrt{121}}{12}$
$\frac{+31 \pm 11}{12}$
$\frac{+31 + 11}{12}$ OR $\frac{+31 - 11}{12}$
$\frac{42}{12}$ OR $\frac{20}{12}$
$\frac{7}{2}$ OR $\frac{5}{3}$
$\therefore S.S = \{\frac{7}{2}, \frac{5}{3}\}$
Verification:
$6(\frac{7}{2})^2 - 31(\frac{7}{2}) = -35$
$\frac{3}{6}(\frac{49}{4}) - \frac{217}{2} = -35$
$\frac{147}{2} - \frac{217}{2} = -35$
$\frac{147 - 217}{2} = -35$
$-35 = -35$
$6(\frac{5}{3})^2 - 31(\frac{5}{3}) = -35$
$\frac{2}{6}(\frac{25}{9}) - \frac{155}{3} = -35$
$\frac{50}{3} - \frac{155}{3} = -35$
$-\frac{105}{3} = -35$
$-35 = -35$

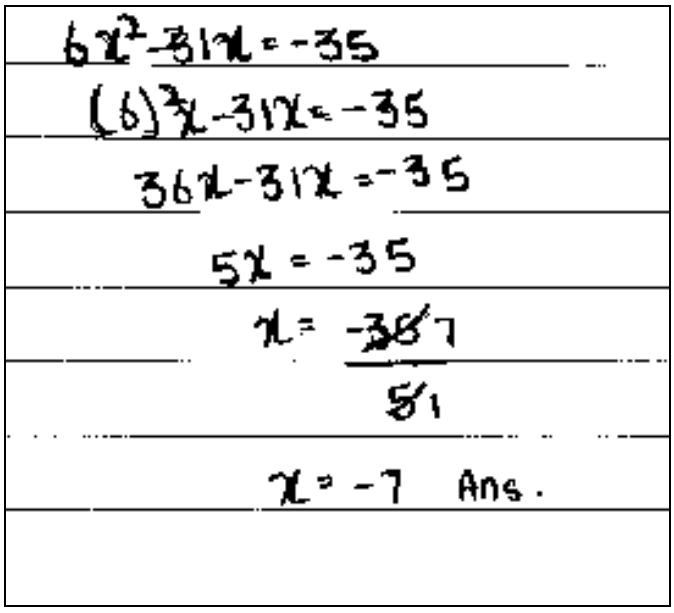
*Weaker responses* indicated that candidates wrote wrong quadratic formula or failed to identify value of *a, b* and *c* correctly. In few other responses, it was noted that candidates correctly identified *a, b* and *c* but failed to apply quadratic formula correctly.

Some candidates used trial and error method to solve the equation but failed to find the solution set of the given quadratic equation. Incorrect use of algebraic operations was evident in many responses.

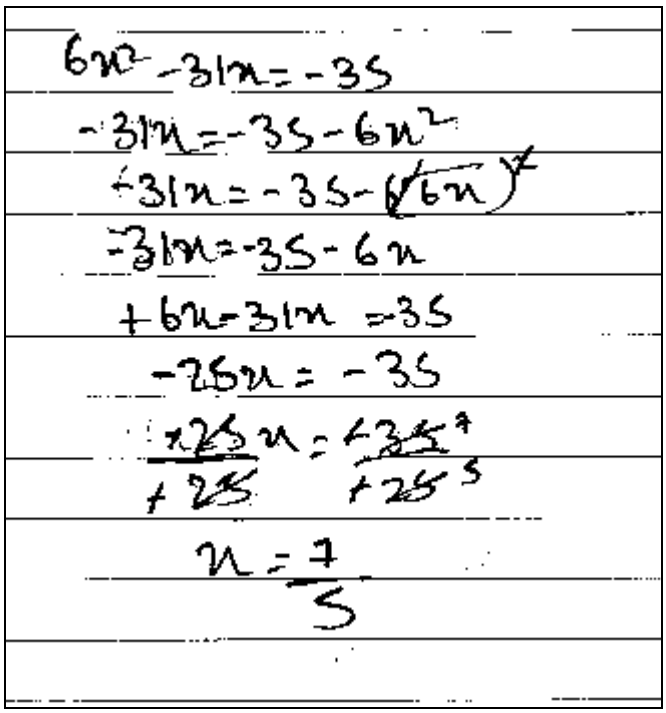
**Example 1:**




**Example 2:**



**Example 3:**



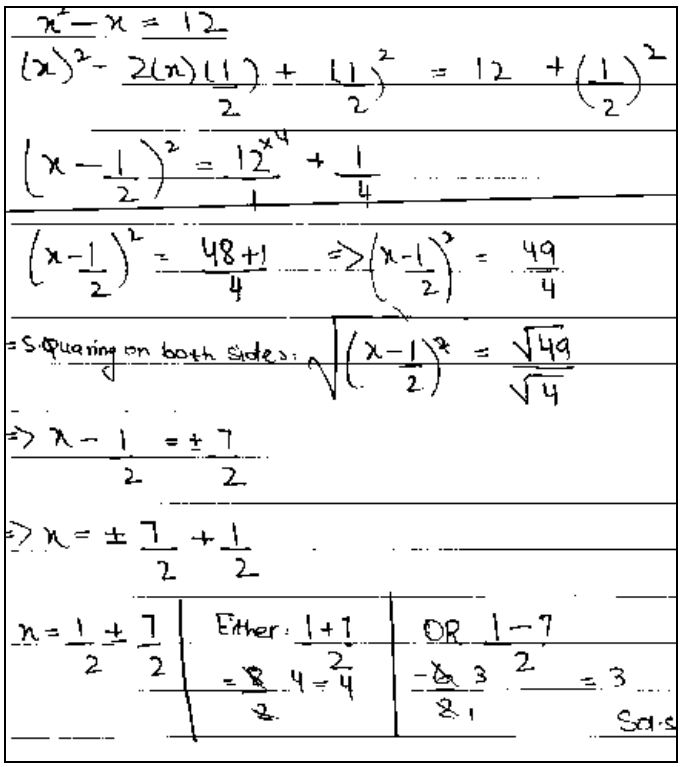

**Question 5b:**

Solve $x^2 - x = 12$ by completing square method.

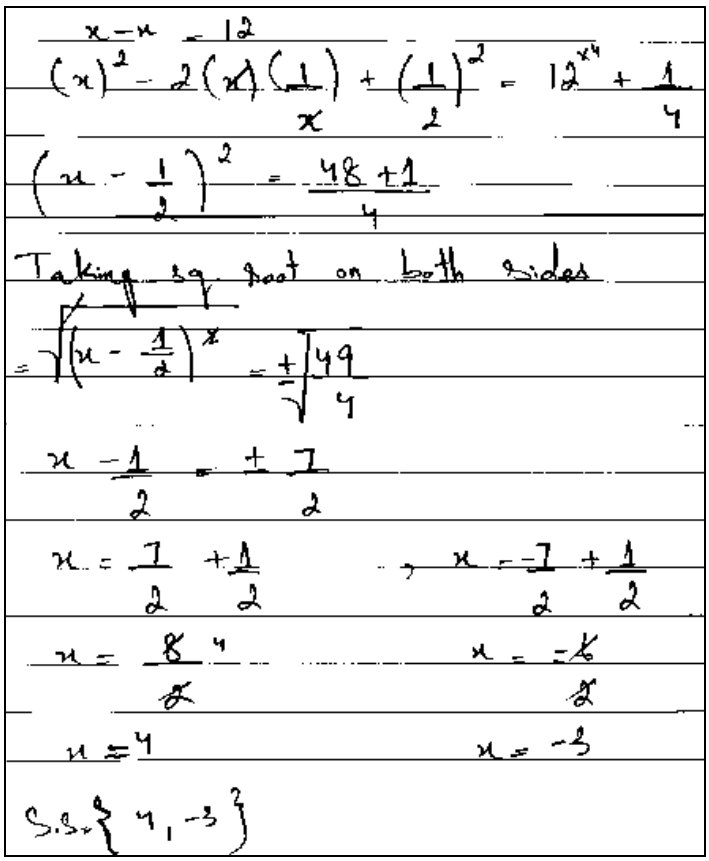
This question was based on the concept of completing square method to solve quadratic equation. Very fewer candidates were able to solve it correctly.

*Better responses* indicated that the candidates were well-versed with the steps involved in the solution of quadratic equation using completing square method. They added the square of half of the coefficient of *x* to both sides of the given quadratic equation to make L.H.S. a complete square. After completing the square, they took the square root on both sides and simplified correctly to find the solution set of the given equation.

**Example 1:**

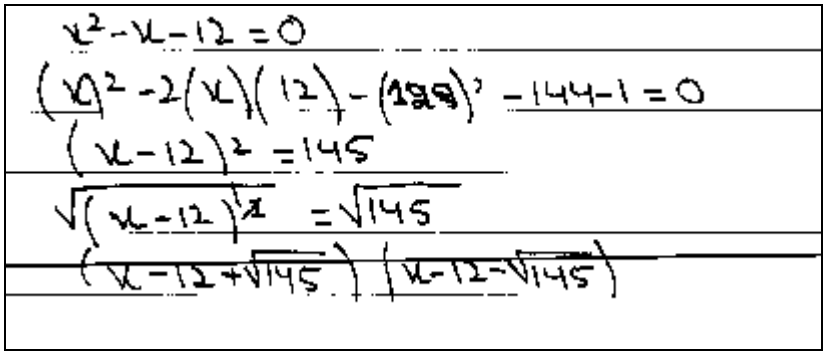



**Example 2:**

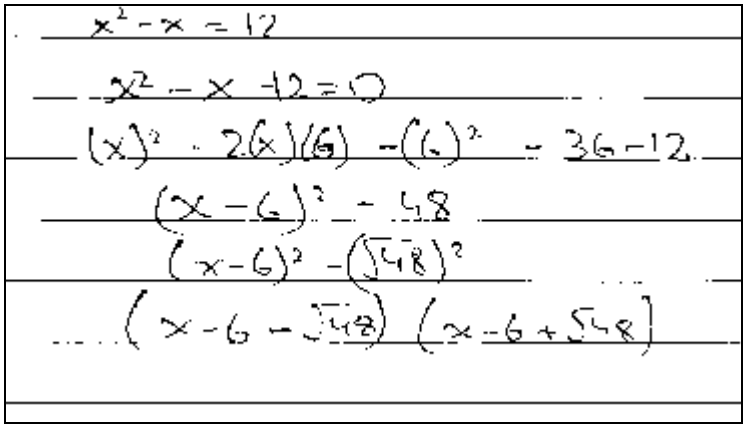


*Weaker response* indicated that candidates solved the question by using other methods to solve quadratic equation. Though, it was clearly mentioned in the question to solve $x^2 - x = 12$ by completing square method. In other responses, it was noted that candidates failed to follow the correct sequence of the steps to solve the equation by using completing square method.

**Example 1:**




**Example 2:**



**Question 6a:**

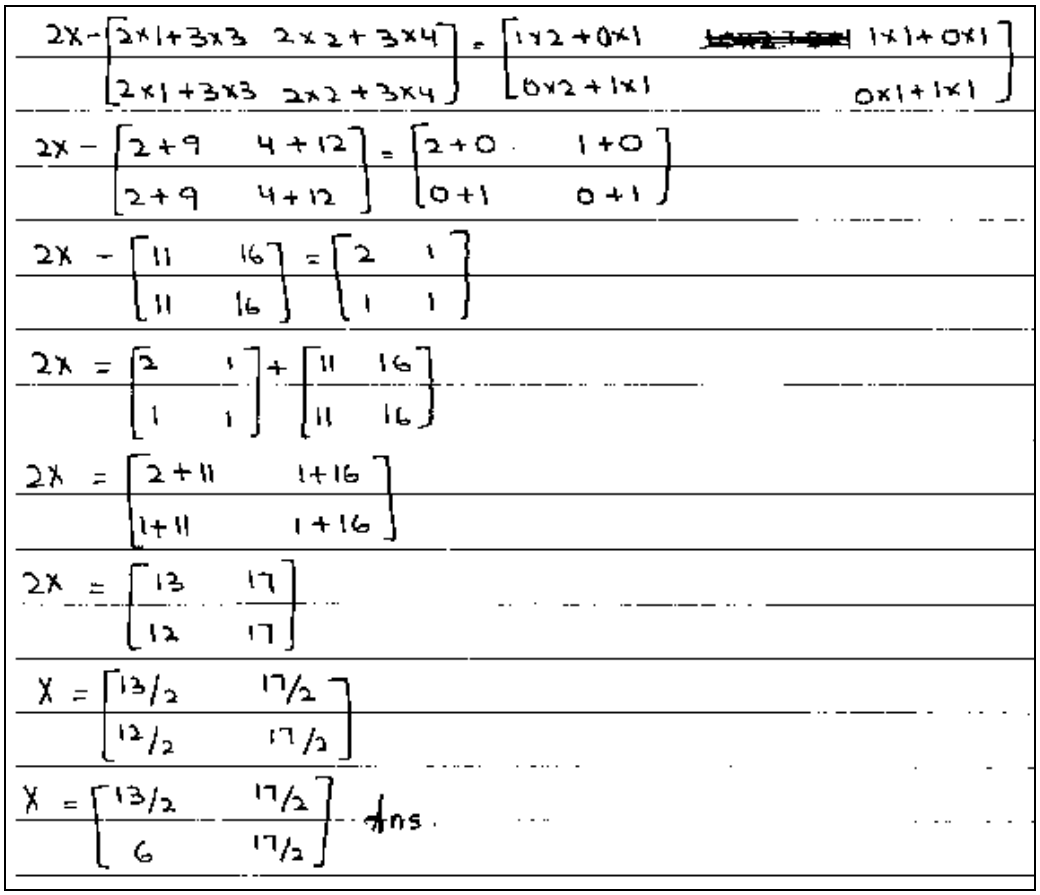
Find the value of matrix $X$ in the following equation.

$$2X - \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

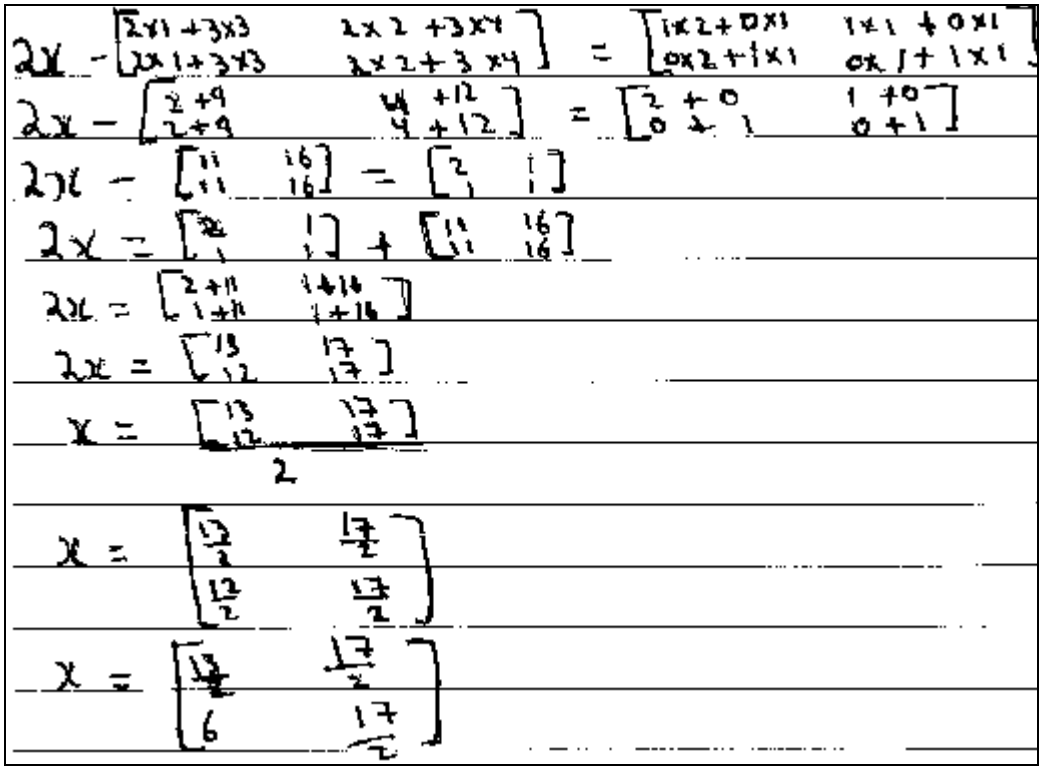
This question was based on the concept of matrix multiplication and addition of matrices.

*Better responses* showed that candidates had good understanding of the concepts of multiplication and addition of matrices; therefore, they were able to find the matrix $X$ from the given matrix equation.

**Example 1:**



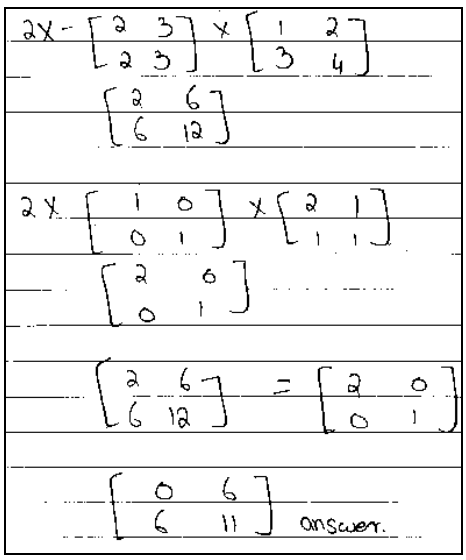
**Example 2:**



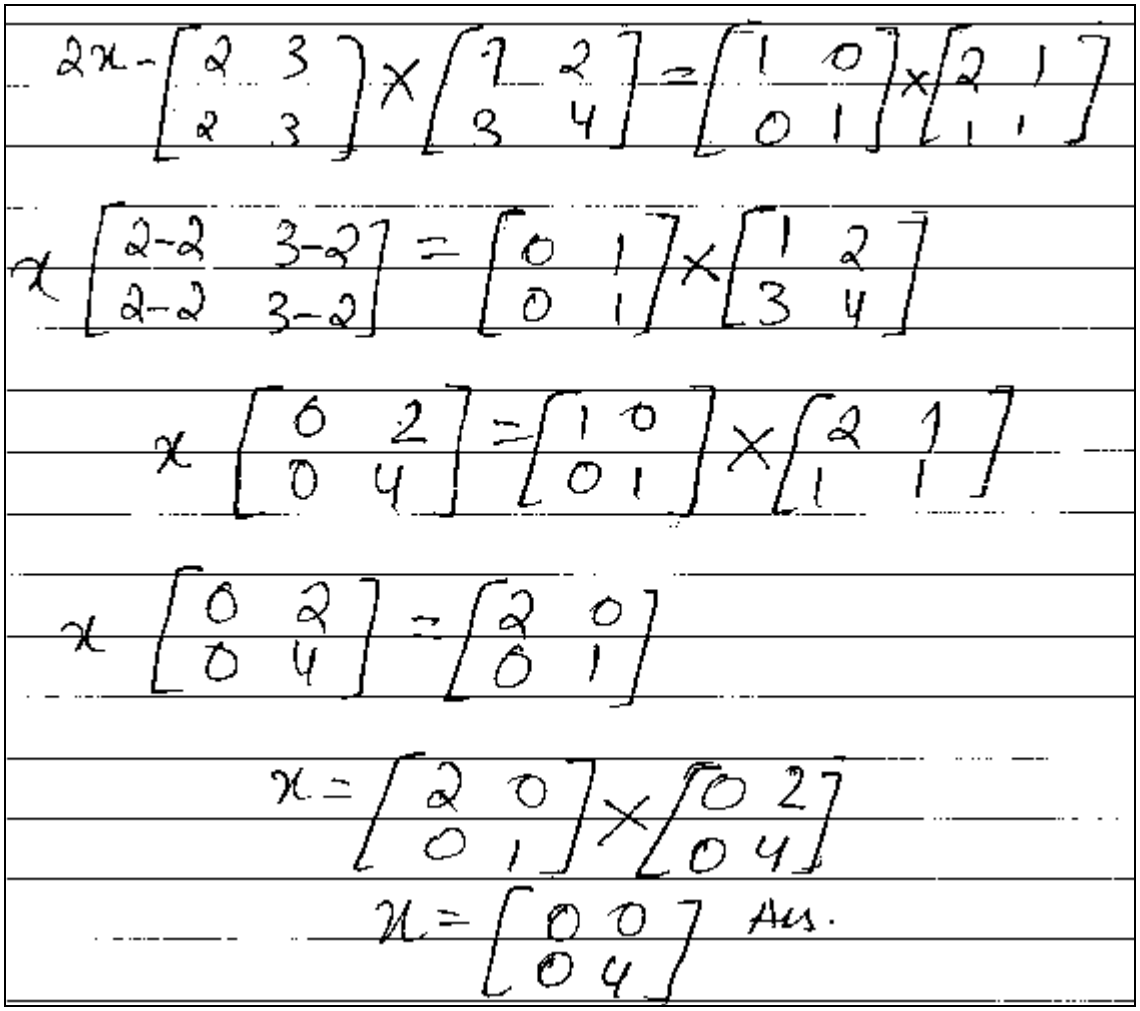
*Weaker responses* exhibited lack of understanding of the concepts of multiplication and addition of matrices and made different types of mistakes. One common mistake noted in the multiplication was as follows:

$$\begin{bmatrix}2 & 3\\ 2 & 3\end{bmatrix}\times\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}2\times1 & 3\times2\\ 2\times3 & 3\times4\end{bmatrix}$$

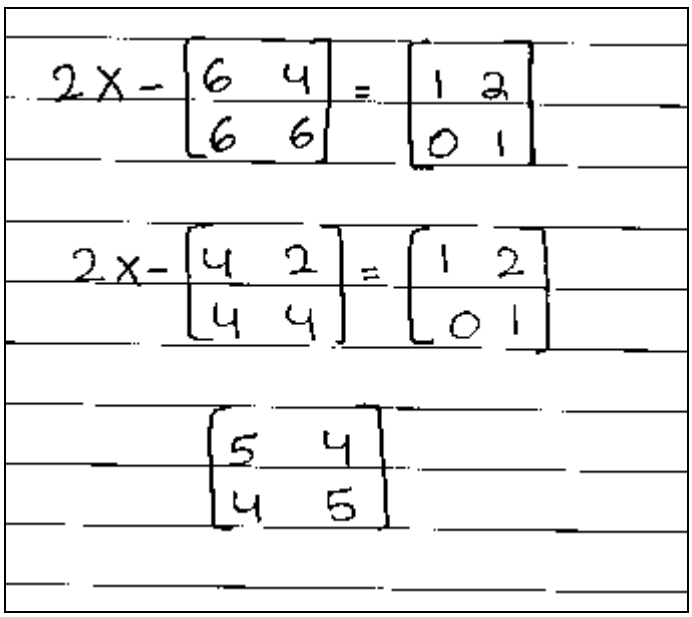
**Example 1:**



**Example 2:**




**Example 3:**



**Question 6b:**

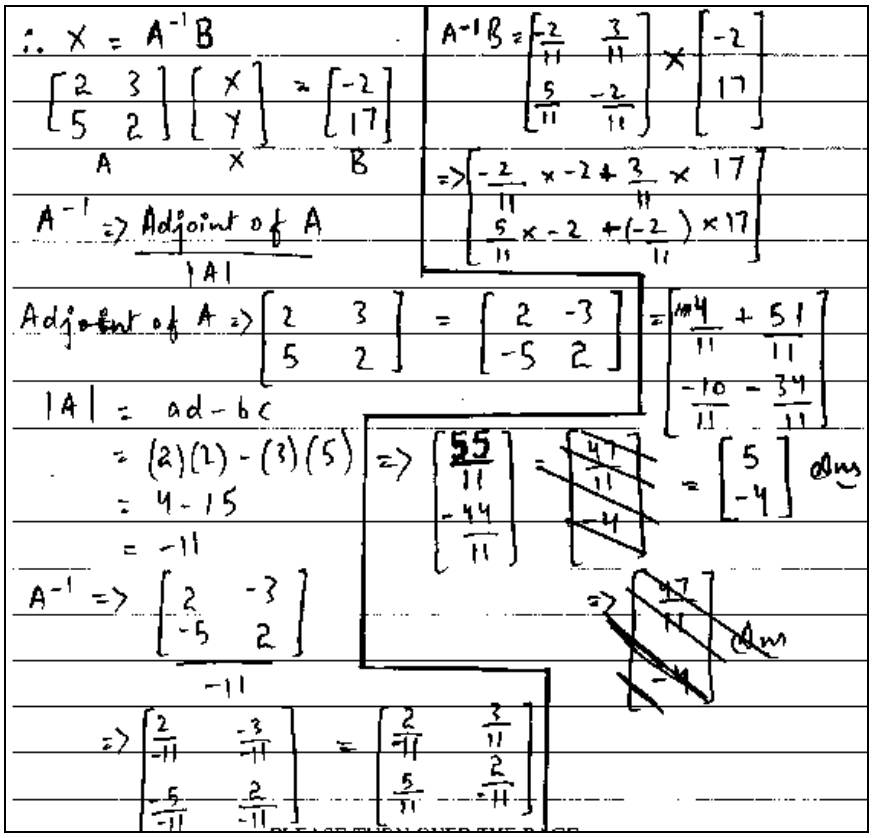
Find the values of *x* and *y* for the given simultaneous linear equations using the inverse matrix method.

$2x+3y=-2$

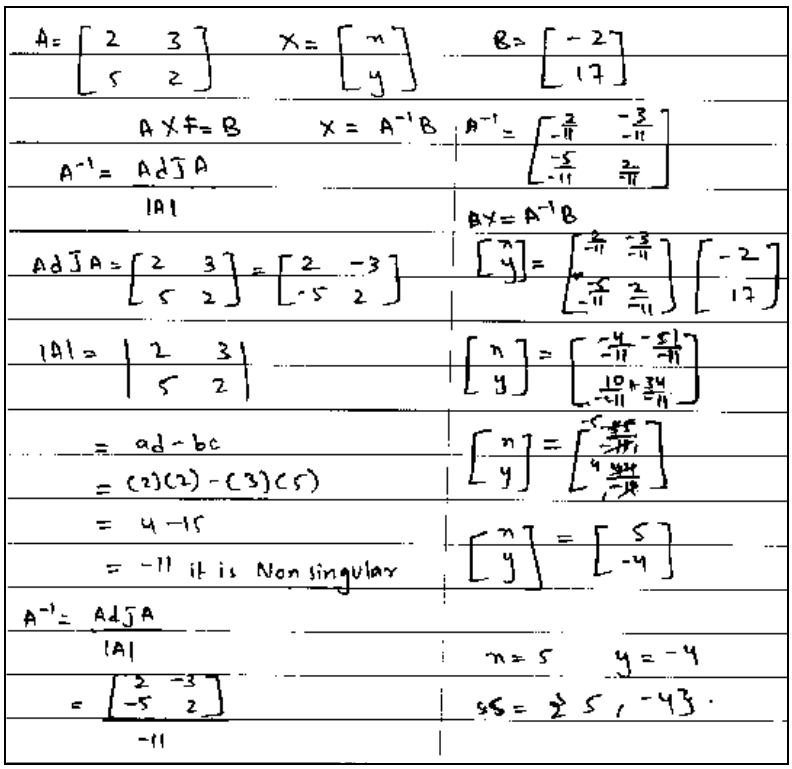
$5x+2y=17$

*Better responses* informed that candidates transformed the given equation to the matrix form followed by finding the correct determinant of the coefficient *x* and *y* and, finally, correctly applied the inverse matrix method to find the values of *x* and *y*.

**Example 1:**



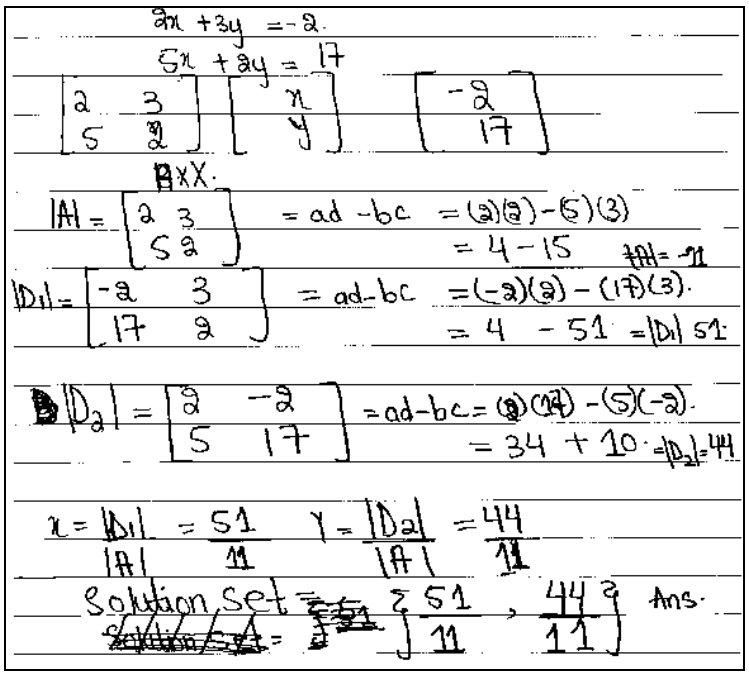
**Example 2:**



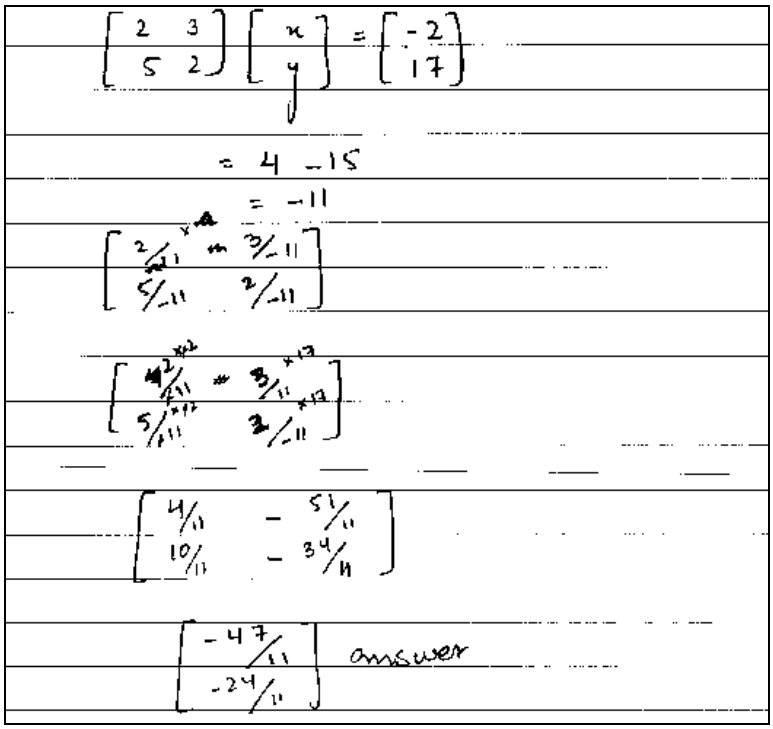

*Weaker responses* showed that candidates made following mistakes:

- Wrong transformation of equations into matrices form
- Failed to calculate value of determinants of the matrix $\begin{bmatrix} 2 & 3 \\ 5 & 2 \end{bmatrix}$
- Failed to find the adjoint and inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 5 & 2 \end{bmatrix}$
- Used Cramer's rule though it was clearly mentioned in the question to apply the inverse matrix method.

**Example 1:**

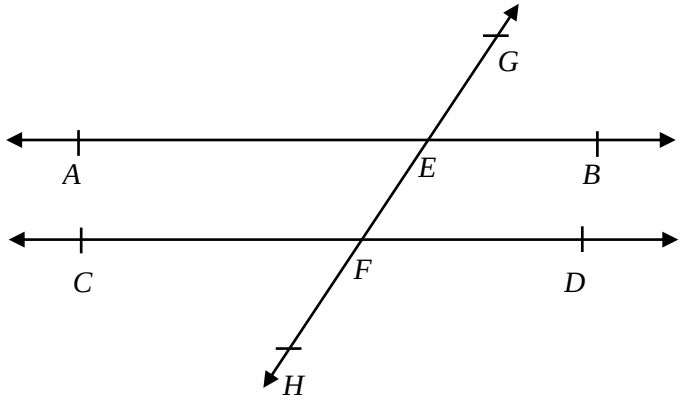



**Example 2:**



**Question 7:**

If in the given diagram, lines *AB* and *CD* are parallel to each other, then complete the following statements.




i. One pair of vertically opposite angles is

ii. One pair of adjacent angles is

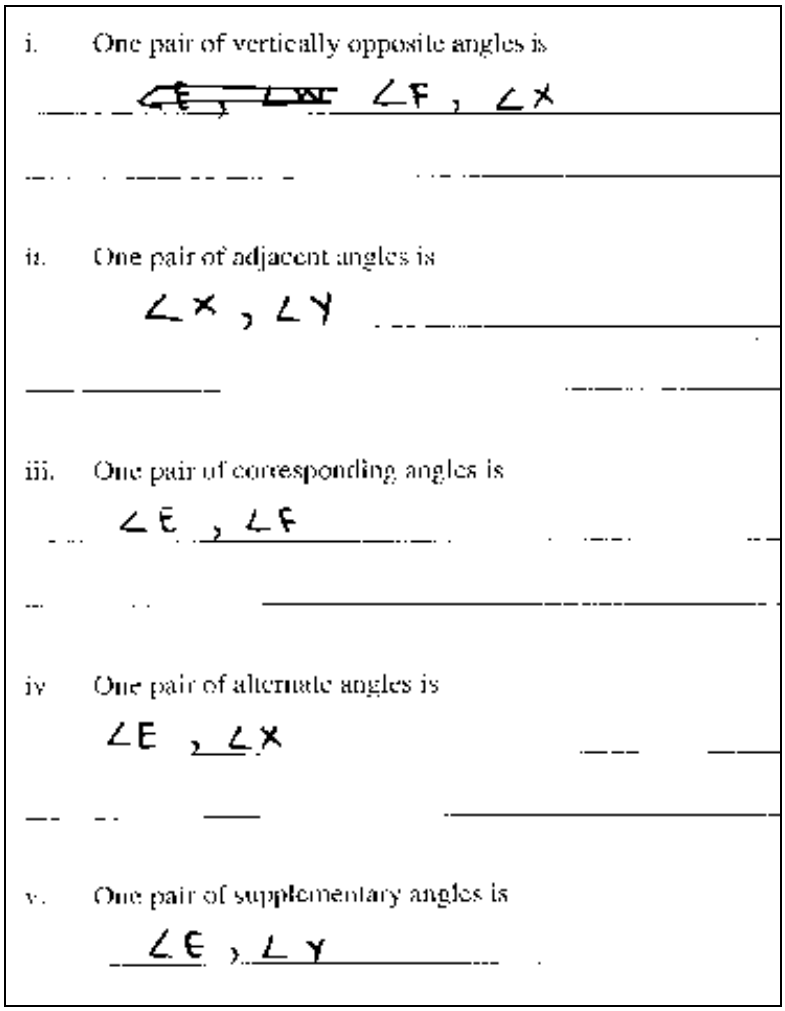
iii. One pair of corresponding angles is

iv. One pair of alternate angles is

v. One pair of supplementary angles is

*Better responses* displayed that candidates were able to comprehend the diagram and were successful in completing the statement testing the understanding of the different types of angles associated with two lines or a transversal intersecting two parallel lines. But one thing worth mentioning here is that many candidates added their own symbols in the diagram to complete the given statements. It shows that there is need to emphasise that how to read and write an angle when its vertex and rays are given, e.g. one pair of vertically opposite angles is $\angle AEG$ and $\angle BEF$.

**Example 1:**



**Example 2:**

i. One pair of vertically opposite angles is

$\angle F$ , $\angle I$

ii. One pair of adjacent angles is

$\angle E$ , $\angle K$

iii. One pair of corresponding angles is

$\angle E$ , $\angle F$

iv. One pair of alternate angles is

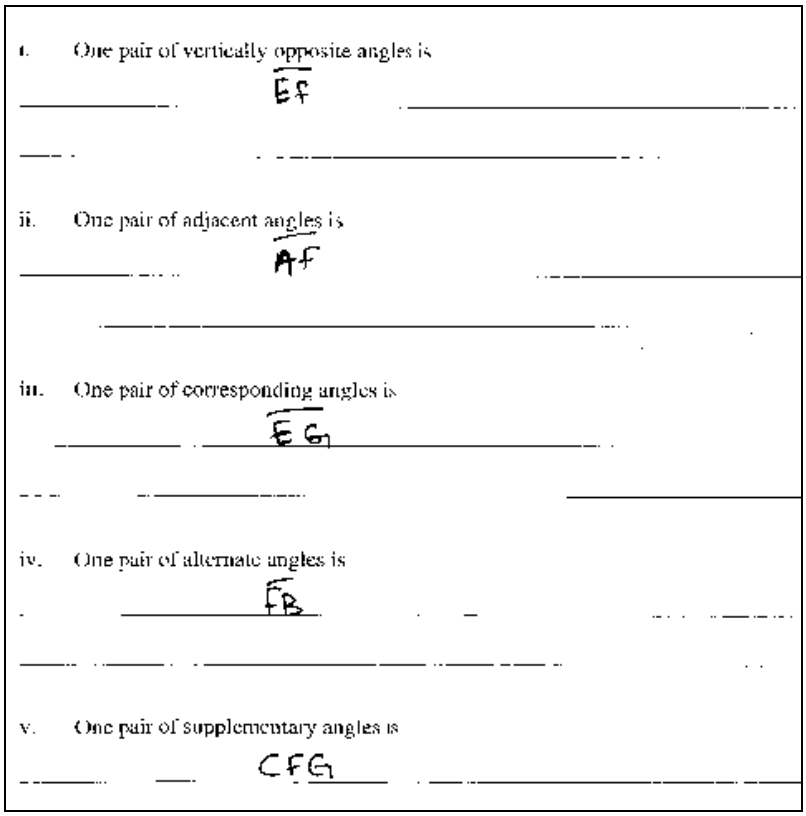
$\angle J$ , $\angle K$

v. One pair of supplementary angles is

$\angle I$ , $\angle K$

*Weaker responses* indicated candidates' lack of understanding of the different types of angles associated with two lines or a transversal intersecting two parallel lines. Hence, they were unable to complete the statement given in the question correctly.

**Example 1:**



**Example 2:**

i. One pair of vertically opposite angles is A and C

ii. One pair of adjacent angles is E and F.

iii. One pair of corresponding angles is C and D

iv. One pair of alternate angles is H and G

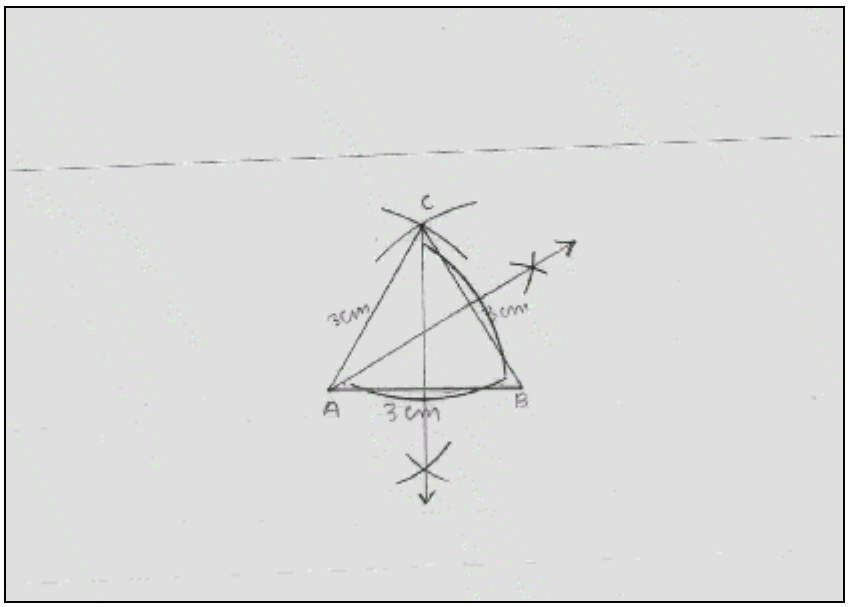
v. One pair of supplementary angles is A and D.

**Question 8**:

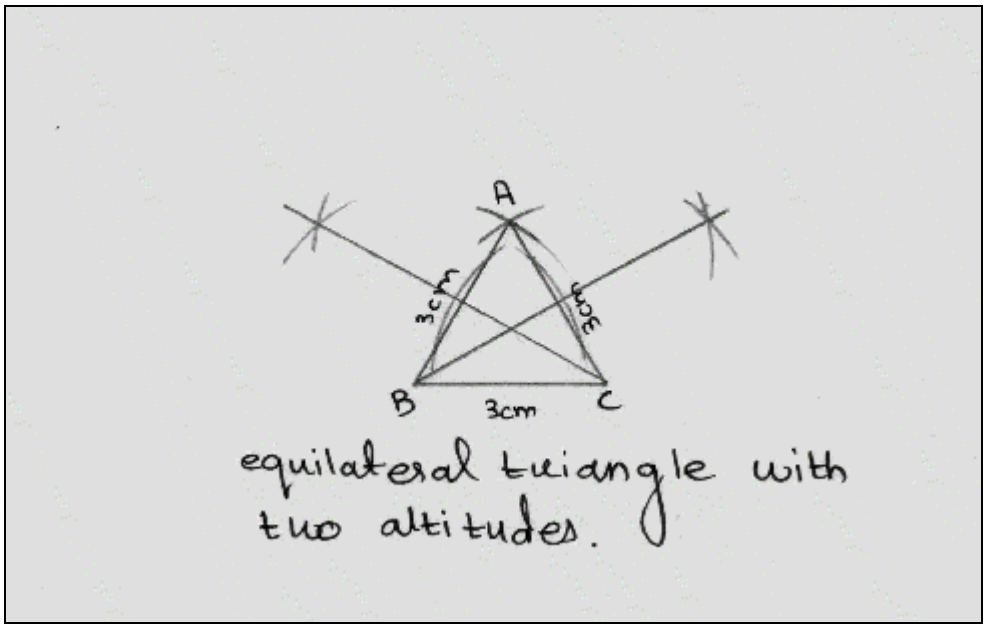
Construct an equilateral triangle *ABC* with each side measuring 3cm. Also draw any two altitudes of triangle *ABC*.

*Better responses* exhibited that candidates have good understanding of the construction of geometrical figures with the given measurements. They also constructed the required altitude correctly.

**Example 1:**

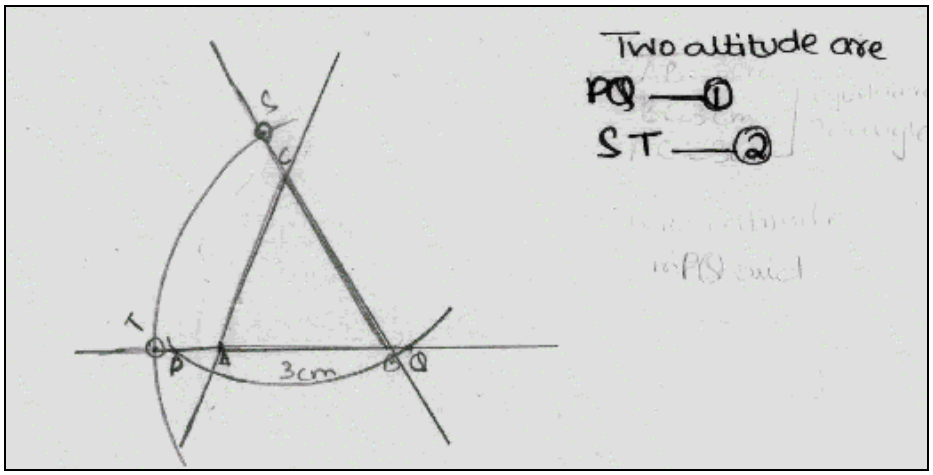



**Example 2:**



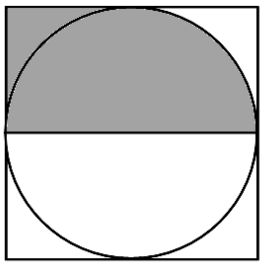
*Weaker responses* displayed that candidates were able to draw the equilateral triangle but failed to draw the altitudes of the triangle.

**Example:**



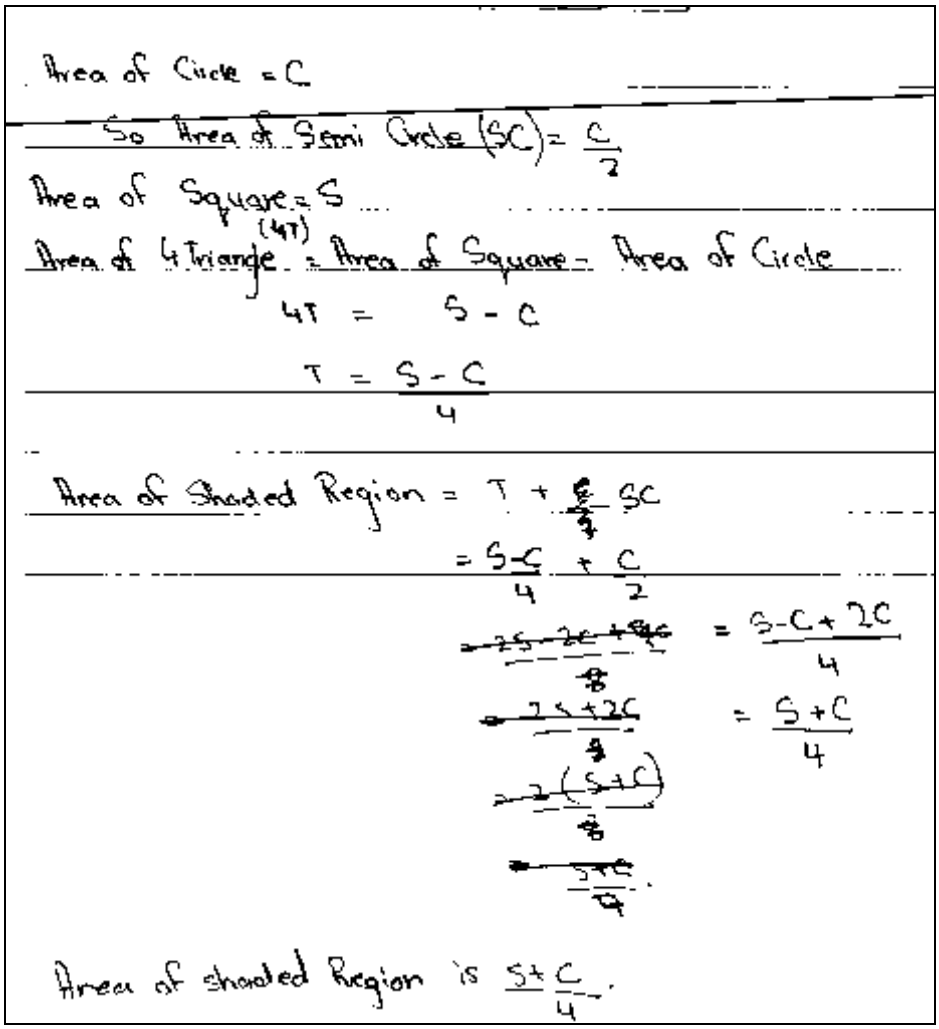
**Question 9:**

In the given diagram, the area of the square is *s* and the area of the circle is *c*. Find the area of the shaded region in the diagram.



*Better response* reported that candidates first found the area of the semi-circle followed by the area of the shaded corner of the square. Finally, they added the area of the semi-circle and area of the shaded corner of the square to find the total area of the shaded region.

**Example 1:**



**Example 2:**

Area of outer side of a circle = Area of square - Area of circle

$= S - C$

Area of shaded region of square $= \frac{S-C}{4}$

Area of shaded region of circle $= \frac{\text{Area of circle}}{2}$

$= \frac{C}{2}$

Area of total shaded region = shaded region of square + shaded region of circle

Area of total shaded region $= \frac{S-C}{4} + \frac{C}{2} \Rightarrow \frac{S-C+2C}{4} \Rightarrow \frac{S+C}{4}$

**Example 3:**

Area of a square without circle $= S - C$

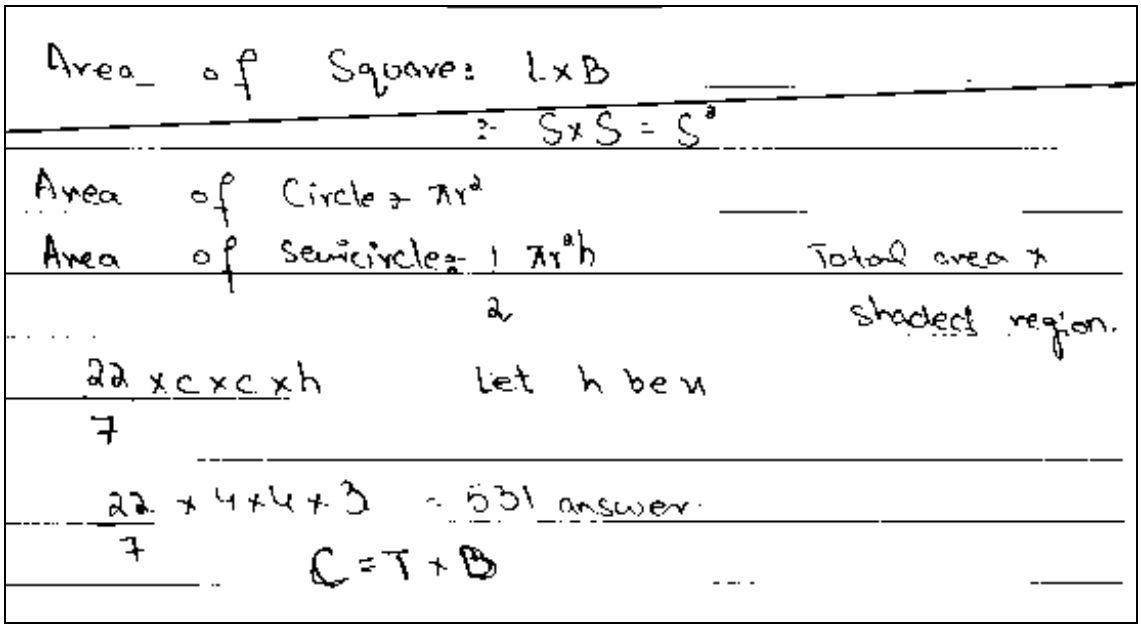
Area of shaded square $= S - C \times \frac{1}{4}$

Area of a shaded circle $= \frac{C}{2}$

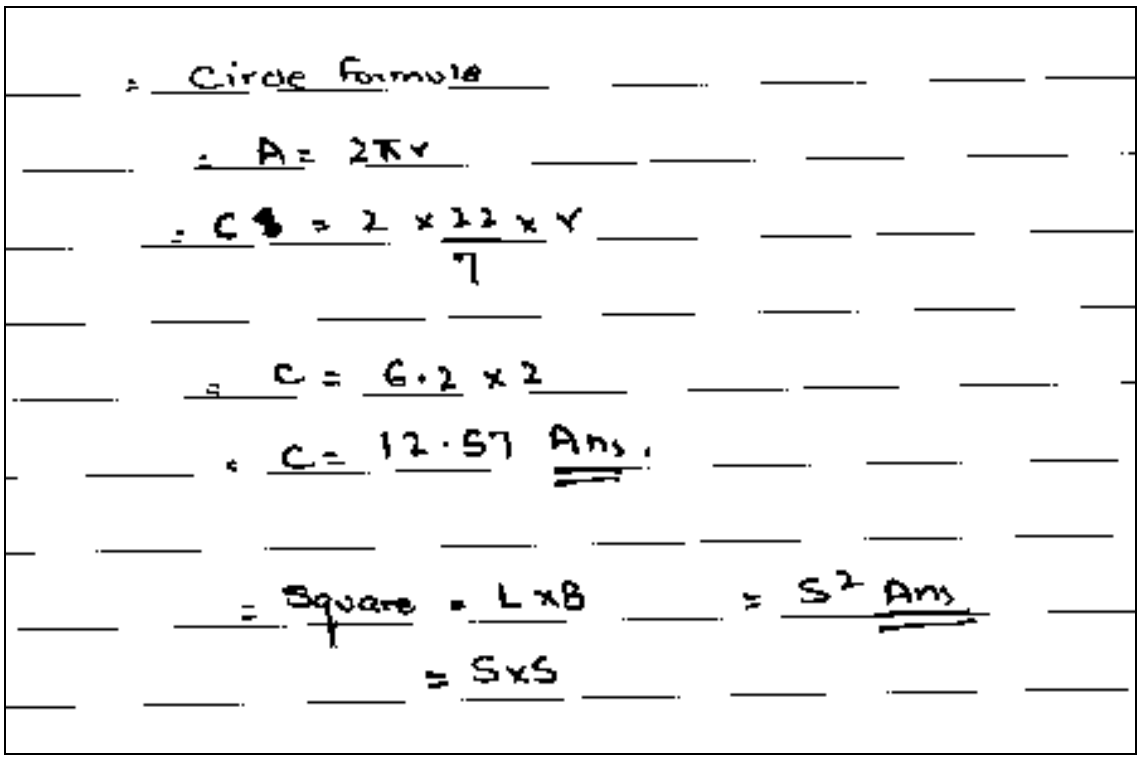
Total shaded region $= \left(S - C \times \frac{1}{4}\right) + \left(\frac{C}{2}\right)$

*Weaker responses* showed that candidates wrote wrong formulae of area of circle and failed to find the correct values for the shaded region. Hence, they were unable to find the required area of the shaded region. The following examples cite different mistakes and misconceptions noted in the weaker responses.

**Example 1:**

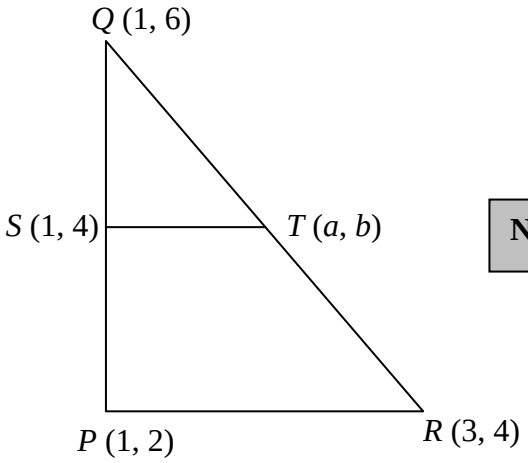



**Example 2:**



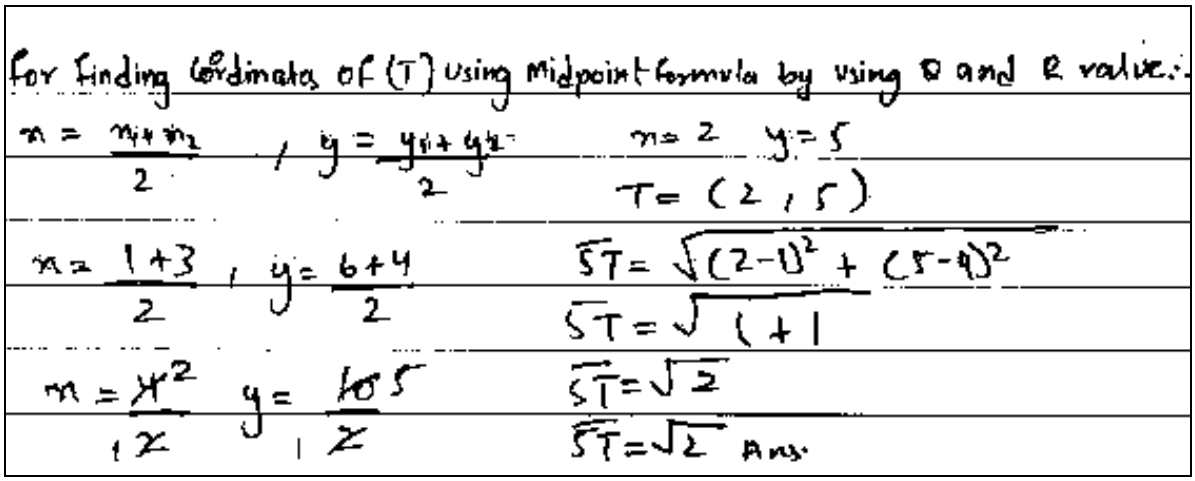
**Question 10:**

The coordinates of the vertices of a triangle *PQR* are *P* (1, 2), *Q* (1, 6) and *R* (3, 4). The midpoints of *PQ* and *QR* are *S* (1, 4) and *T* (*a*, *b*) respectively. Find the coordinates of point *T* and distance between *S* and *T*.

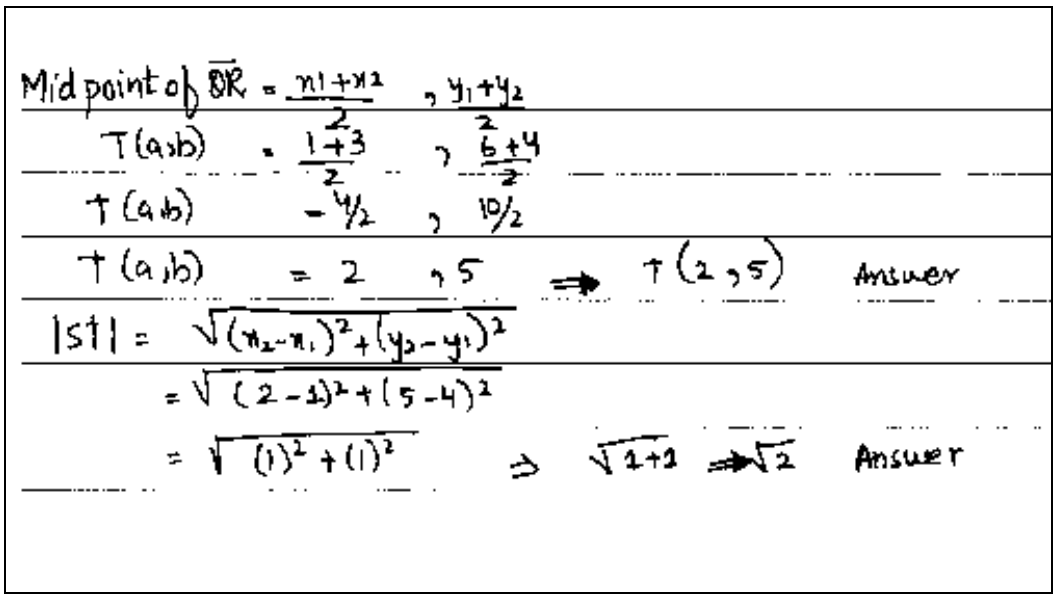



*Better responses* reported that candidates used the mid-point formula to find the vertices of the point *T* followed by the distance formula to find the measurement of the line *ST*.

**Example 1:**



**Example 2:**

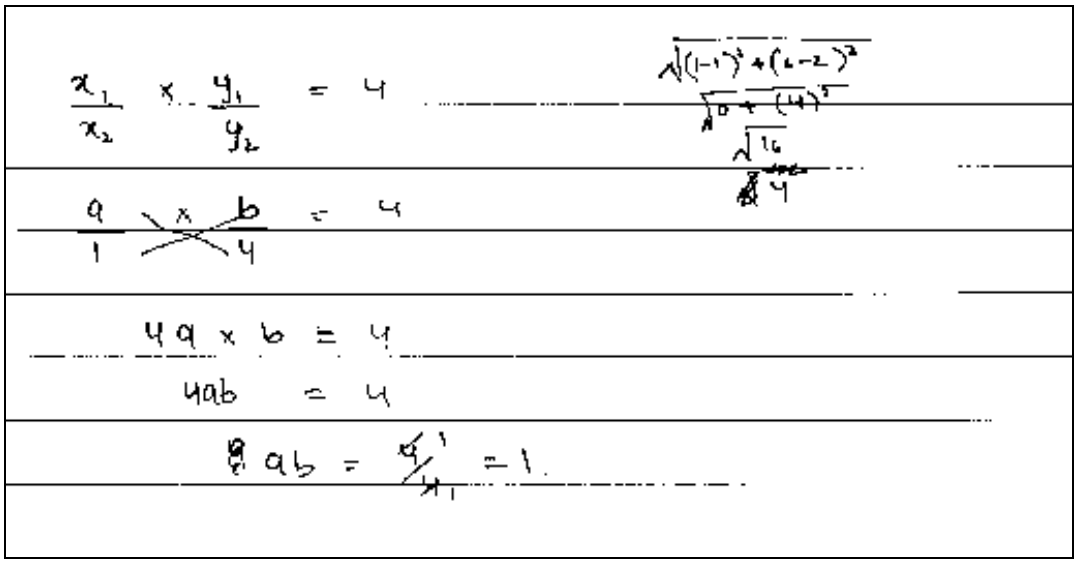



*Weaker responses* showed different mistakes in writing the distance formula and midpoint formula. Some of these are listed below:

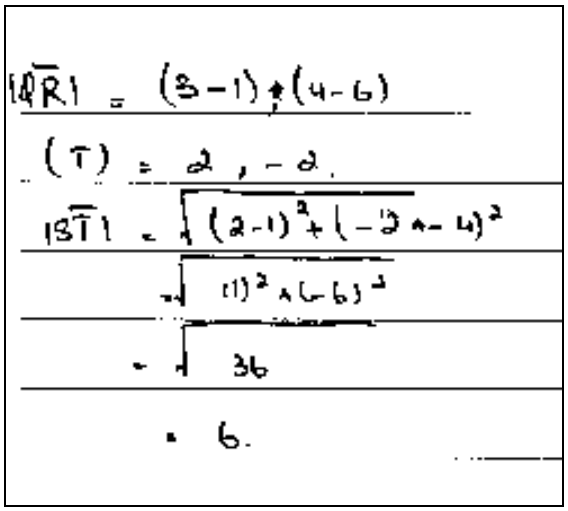
- $d = \sqrt{(x_1 + y_1) + (x_2 + y_2)}$
- $d = \sqrt{(x_1 - y_1) + (x_2 - y_2)}$
- $d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$
- $d = \sqrt{(x_1 + y_1)^2 + (x_2 + y_2)^2}$
- $d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$
- $\text{Mid Point Formula} = \left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$
- $\text{Mid Point Formula} = \left(\frac{x_1 - y_1}{2}, \frac{x_2 - y_2}{2}\right)$
- $\text{Mid Point Formula} = \left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$

The weaker responses also reported that candidates failed to identify the values of $x_1, x_2, y_1$ and $y_2$ correctly. Hence, they failed to fulfil the requirement of the question.

**Example 1:**




**Example 2:**




**Example 3:**