Aga Khan University Examination Board

Notes from E-Marking Centre on SSC-I General Mathematics Examination May 2017

Introduction

This document has been produced for the teachers and candidates of Secondary School Certificate (SSC-I) General Mathematics. It contains comments on candidates' responses to the 2017 SSC-I Examination indicating the quality of the responses and highlighting their relative strengths and weaknesses.

E-Marking Notes

This includes overall comments on candidates' performance on every question and *some* specific examples of candidates' responses which support the mentioned comments. Please note that the descriptive comments represent an overall perception of the better and weaker responses as gathered from the e-marking session. However, the candidates' responses shared in this document represent some specific example(s) of the mentioned comments.

Teachers and candidates should be aware that examiners may ask questions that address the Student Learning Outcomes (SLOs) in a manner that require candidates to respond by integrating knowledge, understanding and application skills they have developed during the course of study. Candidates are advised to read and comprehend each question carefully before writing the response to fulfil the demand of the question.

Candidates need to be aware that the marks allocated to the questions are related to the answer space provided on the examination paper as a guide to the length of the required response. A longer response will not in itself lead to higher marks. Candidates need to be familiar with the command words in the SLOs which contain terms commonly used in examination questions. However, candidates should also be aware that not all questions will start with or contain one of the command words. Words such as 'how', 'why' or 'what' may also be used.

Key observations:

Weaker responses revealed that candidates had problems with conversion of verbal phrases into the mathematical operations to solve problems. In general, questions based on direct and inverse proportion, profit and loss, laws of logarithm and indices were not well attempted. However, questions related to zakat and usher, simple interest, operation on sets, statistics and Venn diagrams were generally well attempted.

Detailed Comments:

Question 1

This was not a well attempted question.

Question 1:

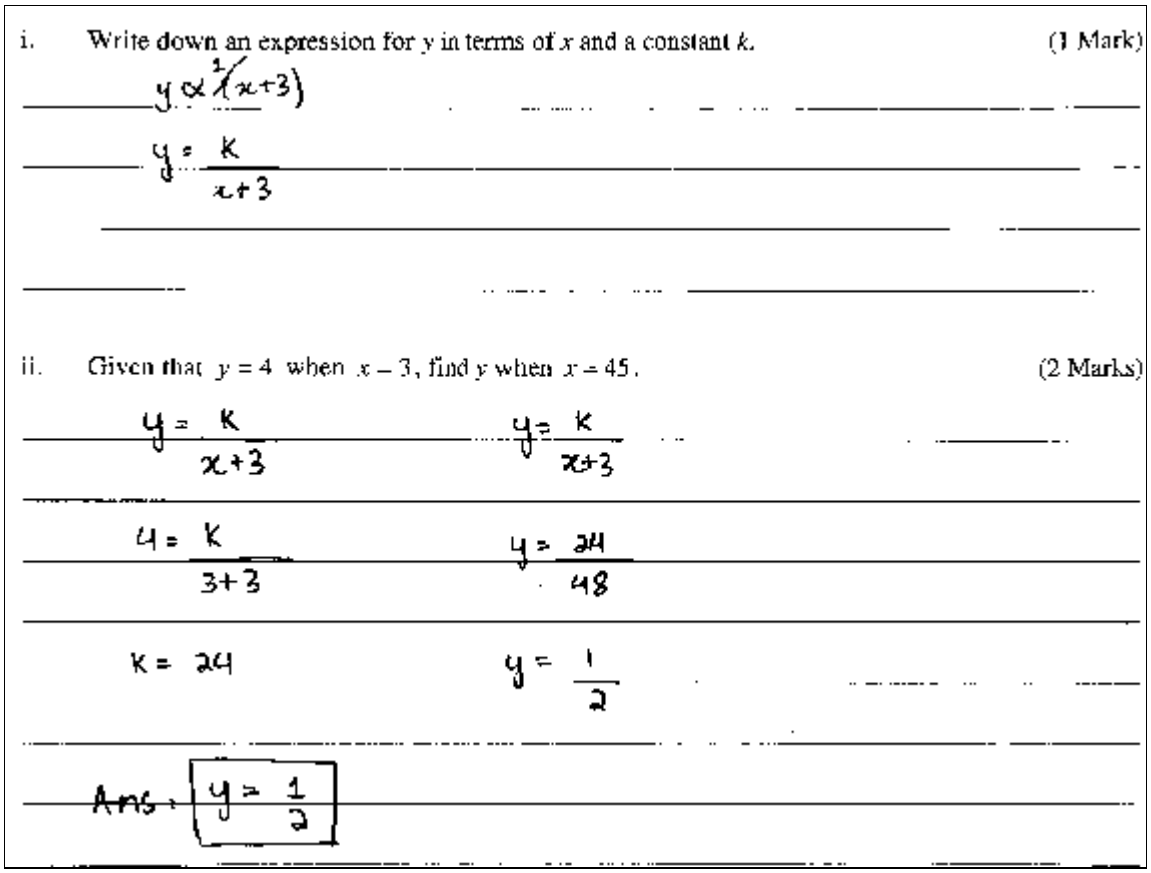
y is inversely proportional to $(x+3)$.

i. Write down an expression for *y* in terms of *x* and a constant *k.*

ii. Given that $y=4$ when $x=3$, find *y* when $x=45$.

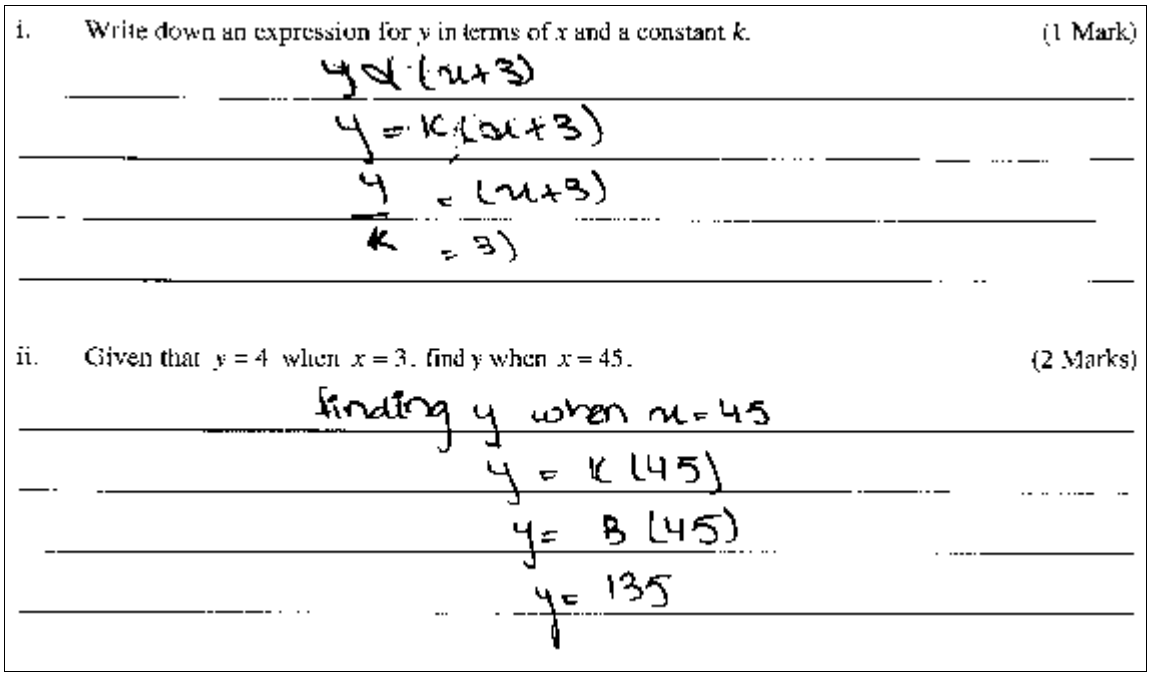
Better responses reflected clear understanding of inverse proportion. Candidates formed the equation $y=\frac{k}{(x+3)}$ successfully. The data given in the question was correctly used to find the value of *k*, which was substituted back in the equation to find *y* when *x* = 45

Example:



Weaker responses reflected that candidates could not translate '*y* is inversely proportional to $(x+3)$'. The most frequent incorrect response was $y = k(x+3)$. Some candidates did not use the given data correctly and made minor mistake in finding the answer.

Example:



Question 2:

This question offered a choice between part a and b. Both parts were attempted equally but the performance in part b was better as compared to part a.

Question 2a:

The total amount of zakat paid by two brothers Ahmar and Bilal is Rs 50,000. If Ahmar paid zakat of Rs 23,000 on his yearly savings, find the

i. amount of zakat paid by Bilal.

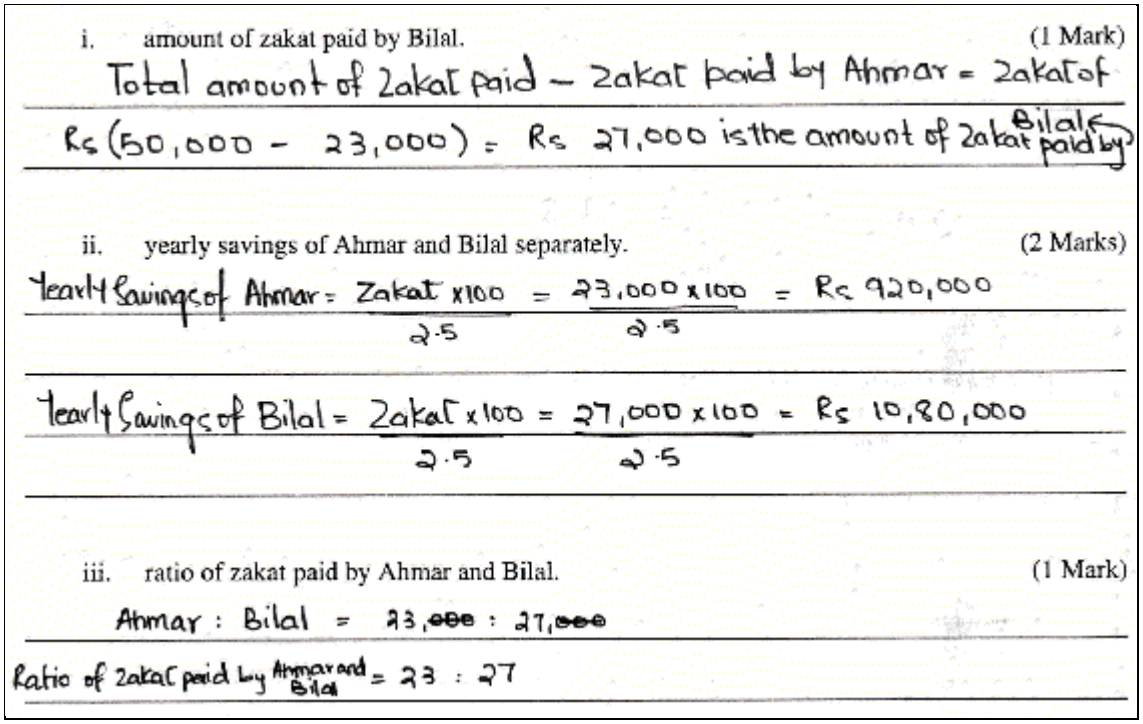
ii. yearly savings of Ahmar and Bilal separately.

iii. ratio of zakat paid by Ahmar and Bilal.

Better responses of part i and ii reflected the understanding of calculation of zakat on yearly savings. Candidates were able to find savings when they were given zakat.

In part iii, candidates exhibited correct application of ratio.

Example:



i. amount of zakat paid by Bilal. (1 Mark)

Total amount of Zakat paid – Zakat paid by Ahmar = Zakat of Bilal is paid by

Rs (50,000 – 23,000) = Rs 27,000 is the amount of Zakat

ii. yearly savings of Ahmar and Bilal separately. (2 Marks)

Yearly Savings of Ahmar = $\frac{\text{Zakat} \times 100}{2.5} = \frac{23,000 \times 100}{2.5}$ = Rs 920,000

Yearly Savings of Bilal = $\frac{\text{Zakat} \times 100}{2.5} = \frac{27,000 \times 100}{2.5}$ = Rs 10,80,000

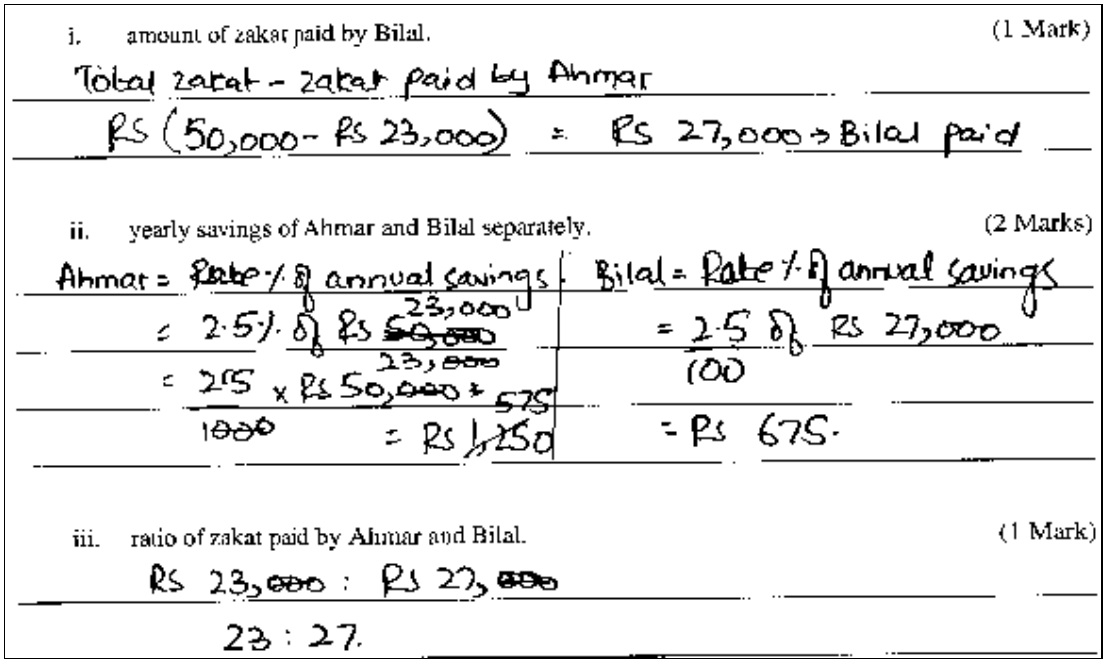
iii. ratio of zakat paid by Ahmar and Bilal. (1 Mark)

Ahmar : Bilal = 23,000 : 27,000

Ratio of Zakat paid by Ahmar and Bilal = 23 : 27

Weaker responses exhibited that candidates did not understand how to use the given information to extract the answer. Many of them assumed that the values given in the question is the annual saving instead of the amount of zakat. This also shows their weak understanding of percentage.

Example:


i. amount of zakat paid by Bilal. (1 Mark)

Total zakat - zakat paid by Ahmar

Rs (50,000 - Rs 23,000) = Rs 27,000 → Bilal paid

ii. yearly savings of Ahmar and Bilal separately. (2 Marks)

Ahmar = Rate % of annual savings

= 2.5% of Rs 23,000

= 2.5 / 100 × Rs 23,000

= 2.5 × Rs 23,000 ÷ 100

= Rs 575

Bilal = Rate % of annual savings

= 2.5 of Rs 27,000

= Rs 675.

iii. ratio of zakat paid by Ahmar and Bilal. (1 Mark)

Rs 23,000 : Rs 27,000

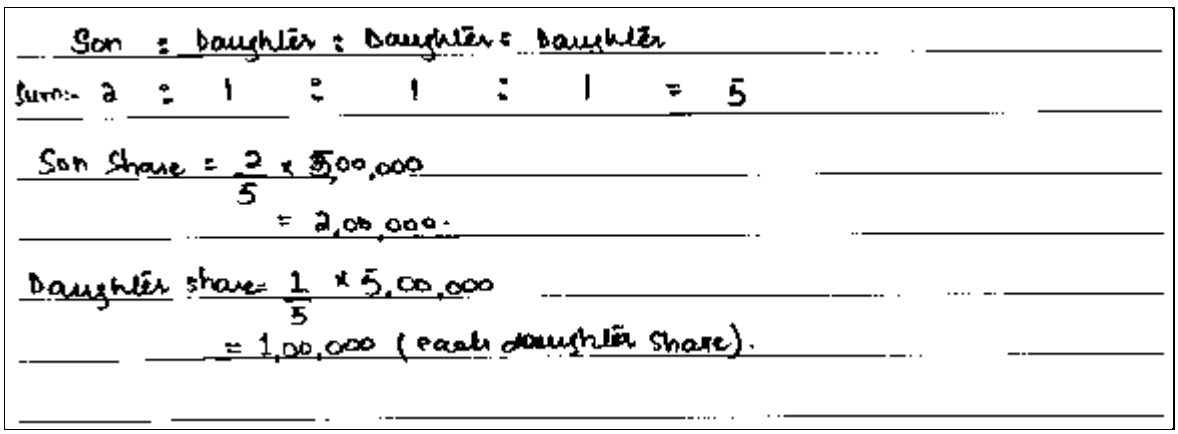
23 : 27.

Question 2b:

Shahid possesses a property worth Rs 500,000 and wishes to distribute his property among his legal heirs that include 1 son and 3 daughters. The son gets two times as much as a daughter gets. Find the amount distributed to each legal heir.

Better responses reflected that candidates could easily understand that the required fractions were ¼ and ¾ which were used to find the answer.

Example:


Son : daughter : Daughter : Daughter

Sum: 2 : 1 : 1 : 1 = 5

Son Share = 2/5 × 5,00,000

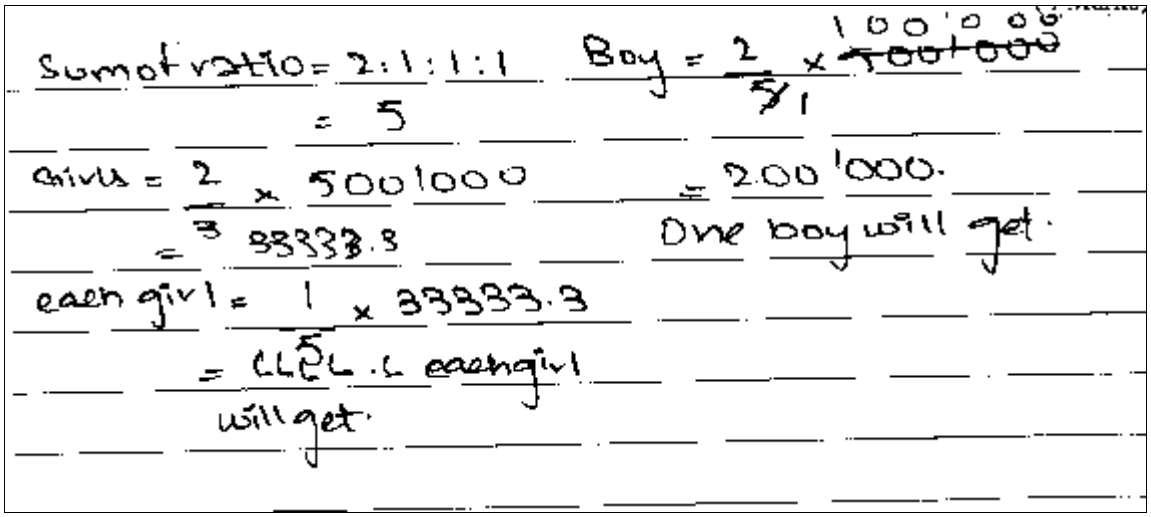
= 2,00,000.

Daughter share = 1/5 × 5,00,000

= 1,00,000 (each daughter share).

Weaker responses reflected that candidates had difficulty deciding the ratio because they could not express the condition 'The son gets twice as much as a daughter gets' mathematically. Most of the candidates found the correct ratio for son but could not find the correct ratio for daughters.

Example:



Question 3:

This question offered a choice between part a and b. Both parts were equally attempted and the performance in both parts was average.

Question 3a:

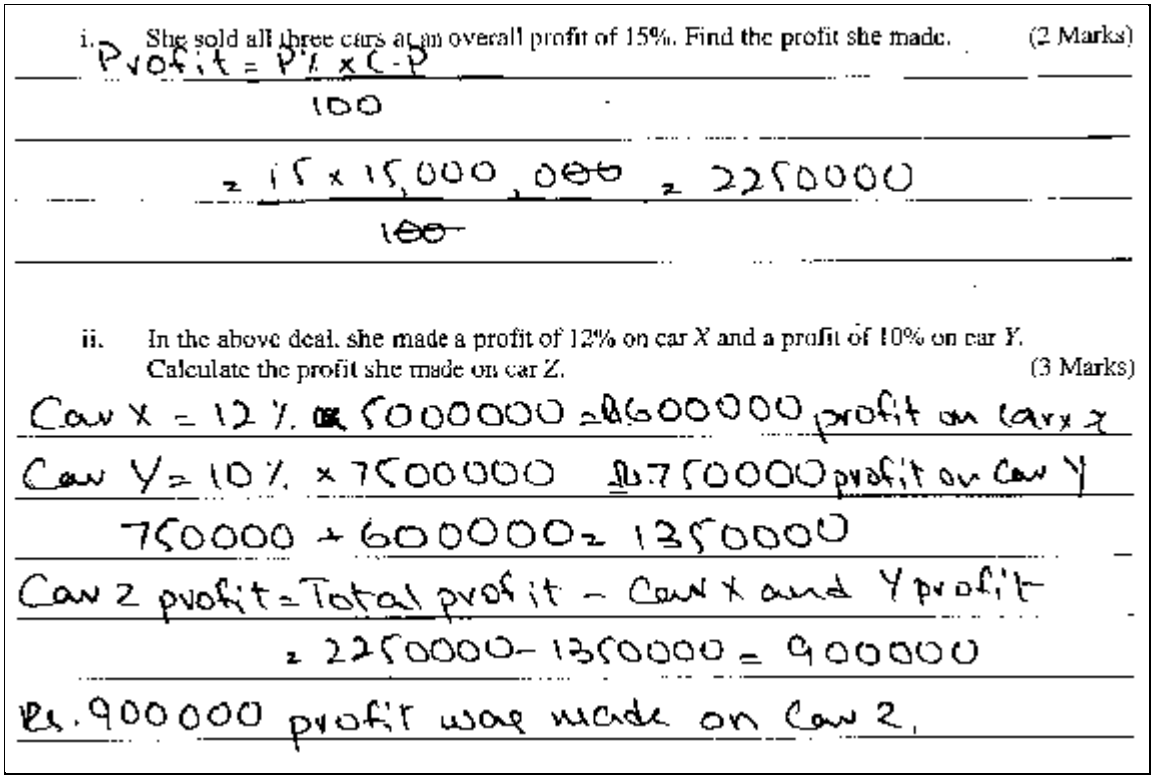
Rabia bought three cars *X*, *Y* and *Z* for a total of Rs 15,000,000. The prices for *X*, *Y* and *Z* are Rs 5,000,000, Rs 7,500,000 and Rs 2,500,000 respectively.

i. She sold all three cars at an overall profit of 15%. Find the profit she made.

ii. In the above deal, she made a profit of 12% on car *X* and a profit of 10% on car *Y*. Calculate the profit she made on car *Z*.

Better responses reflected that the candidates, who correctly translated the given condition, mathematically were able to get the solution. In part i, the prices of *X*, *Y* and *Z* were added and then 15% of profit was found. In part ii, the individual profit on *X* and *Y* was calculated, and subtracted from the total profit to find the profit on *Z*.

Example:


i. She sold all three cars at an overall profit of 15%. Find the profit she made. (2 Marks)

Profit = P% x C-P / 100

= 15 x 15,000,000 / 100 = 2250000

ii. In the above deal, she made a profit of 12% on car *X* and a profit of 10% on car *Y*. Calculate the profit she made on car *Z*. (3 Marks)

Car X = 12% x 5000000 = 600000 profit on car x

Car Y = 10% x 7500000 = 750000 profit on Car Y

750000 + 600000 = 1350000

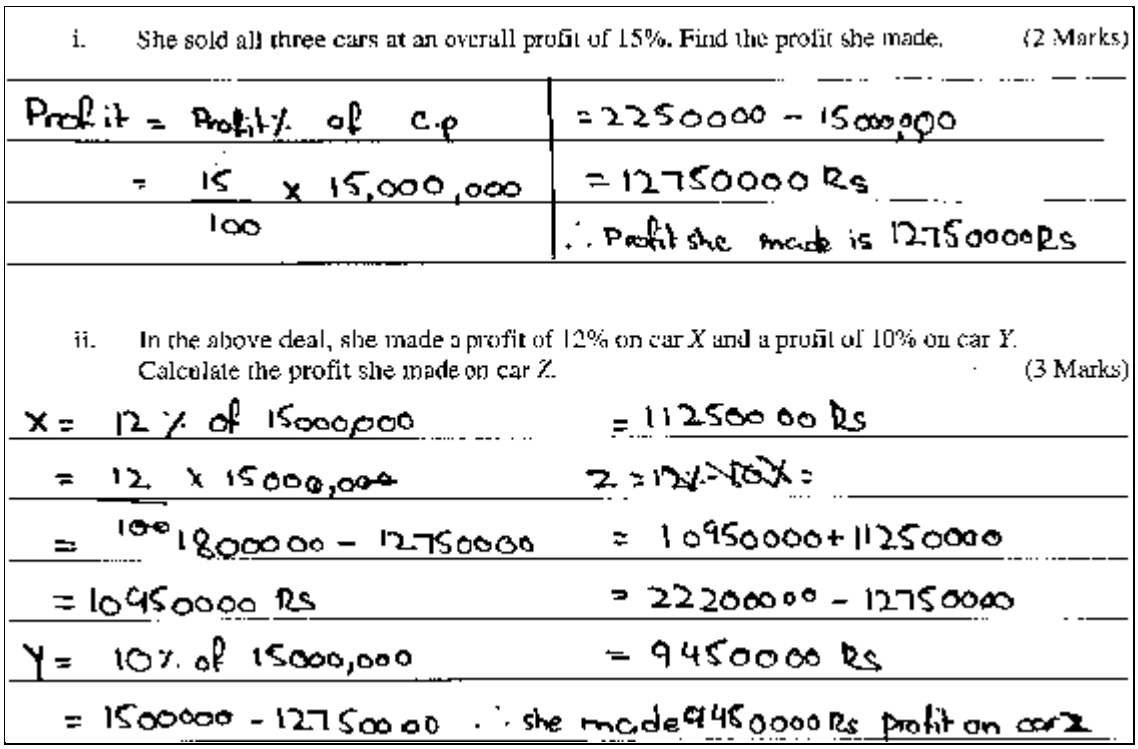
Car 2 profit = Total profit - Car X and Y profit

= 2250000 - 1350000 = 900000

Rs. 900000 profit was made on Car 2.

Weaker responses reflected that most of the candidates used incorrect mathematical models to find the answer. While most of the candidates were able to solve part i, errors were frequent in part ii while finding the profit on car *X* and *Y*. Candidates did guess work after it and could not work out how to find the profit on *Z*.

Example:



i. She sold all three cars at an overall profit of 15%. Find the profit she made. (2 Marks)

Profit = Profit% of C.P

= 15/100 x 15,000,000

= 2250000 − 1500000

= 12750000 Rs

∴ Profit she made is 12750000 Rs

ii. In the above deal, she made a profit of 12% on car *X* and a profit of 10% on car *Y*. Calculate the profit she made on car *Z*. (3 Marks)

X = 12% of 15000000 = 1125000 Rs

= 12/100 x 15000000

= 1800000 − 1275000

= 10950000 Rs

Z = 12% + 10% =

= 10950000 + 1125000

= 22200000 − 12750000

Y = 10% of 15000000 = 9450000 Rs

= 1500000 − 1275000 ∴ she made 9450000 Rs profit on car Z

Question 3b:

The price of each scoop at an ice cream parlour is Rs 85. They offer the following discounts.

DISCOUNT OFFER 1:
BUY A CUP OF TWO SCOOPS, GET 10% OFF
DISCOUNT OFFER 2:
BUY A CUP OF THREE SCOOPS, GET 20% OFF

If Ahad, Waqar and Nasir buy a cup of 3 scoops, 2 scoops, 1 scoop respectively from this parlour, then find the

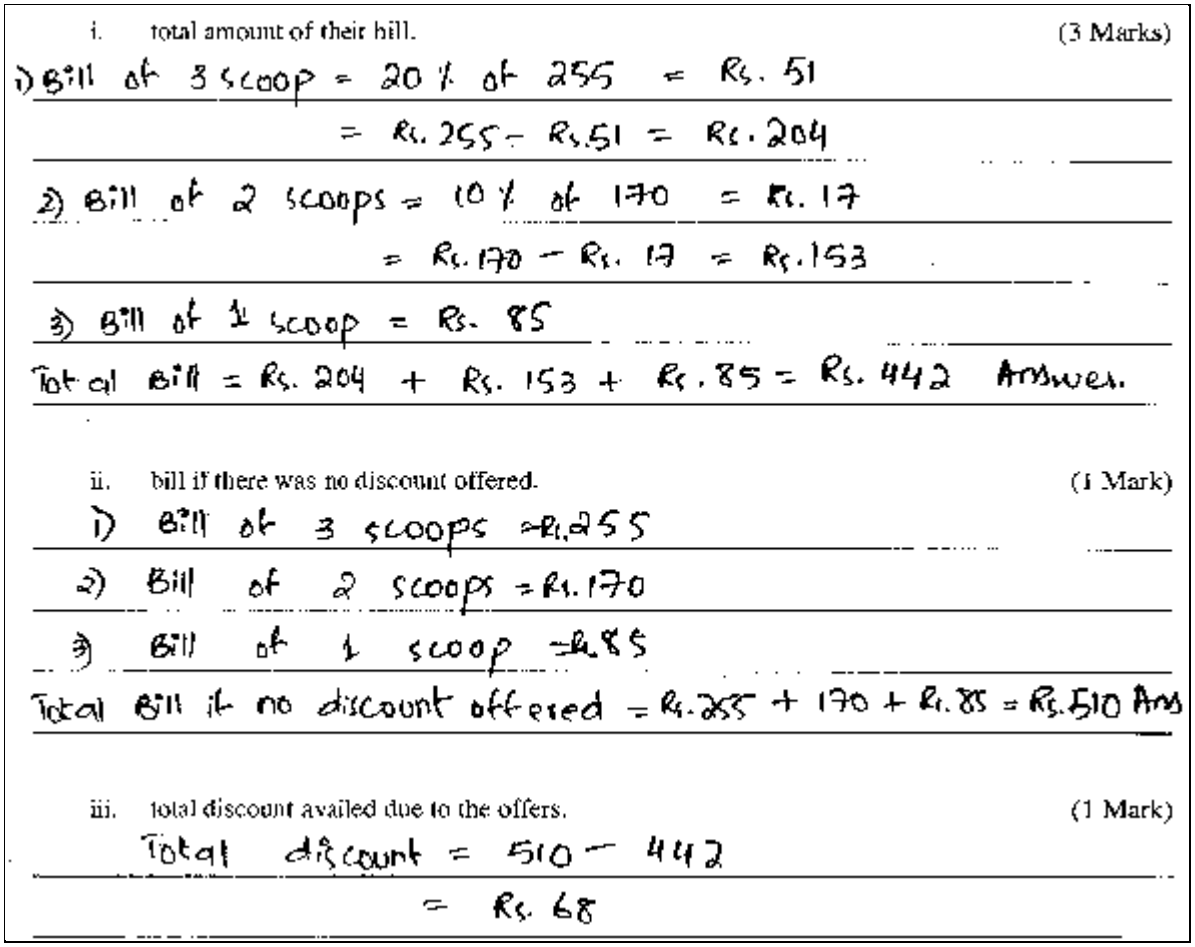
i. total amount of their bill.

ii. bill if there was no discount offered.

iii. total discount availed due to the offers.

Better responses reflected that candidates were able to understand the given discount offers and how the three friends used these offers in their order. They could easily find the total amount of bill as per the given discount offers, i.e. Waqar availed discout offer 1, Ahad availed discount offer 2 while Nasir did not avail any offer so he got no discount.

Example:



i. total amount of their bill. (3 Marks)

1) Bill of 3 scoop = 20 % of 255 = Rs. 51

= Rs. 255 - Rs. 51 = Rs. 204

2) Bill of 2 scoops = 10 % of 170 = Rs. 17

= Rs. 170 - Rs. 17 = Rs. 153

3) Bill of 1 scoop = Rs. 85

Total Bill = Rs. 204 + Rs. 153 + Rs. 85 = Rs. 442 Answer.

ii. bill if there was no discount offered. (1 Mark)

1) Bill of 3 scoops = Rs. 255

2) Bill of 2 scoops = Rs. 170

3) Bill of 1 scoop = Rs. 85

Total Bill if no discount offered = Rs. 255 + 170 + Rs. 85 = Rs. 510 Ans

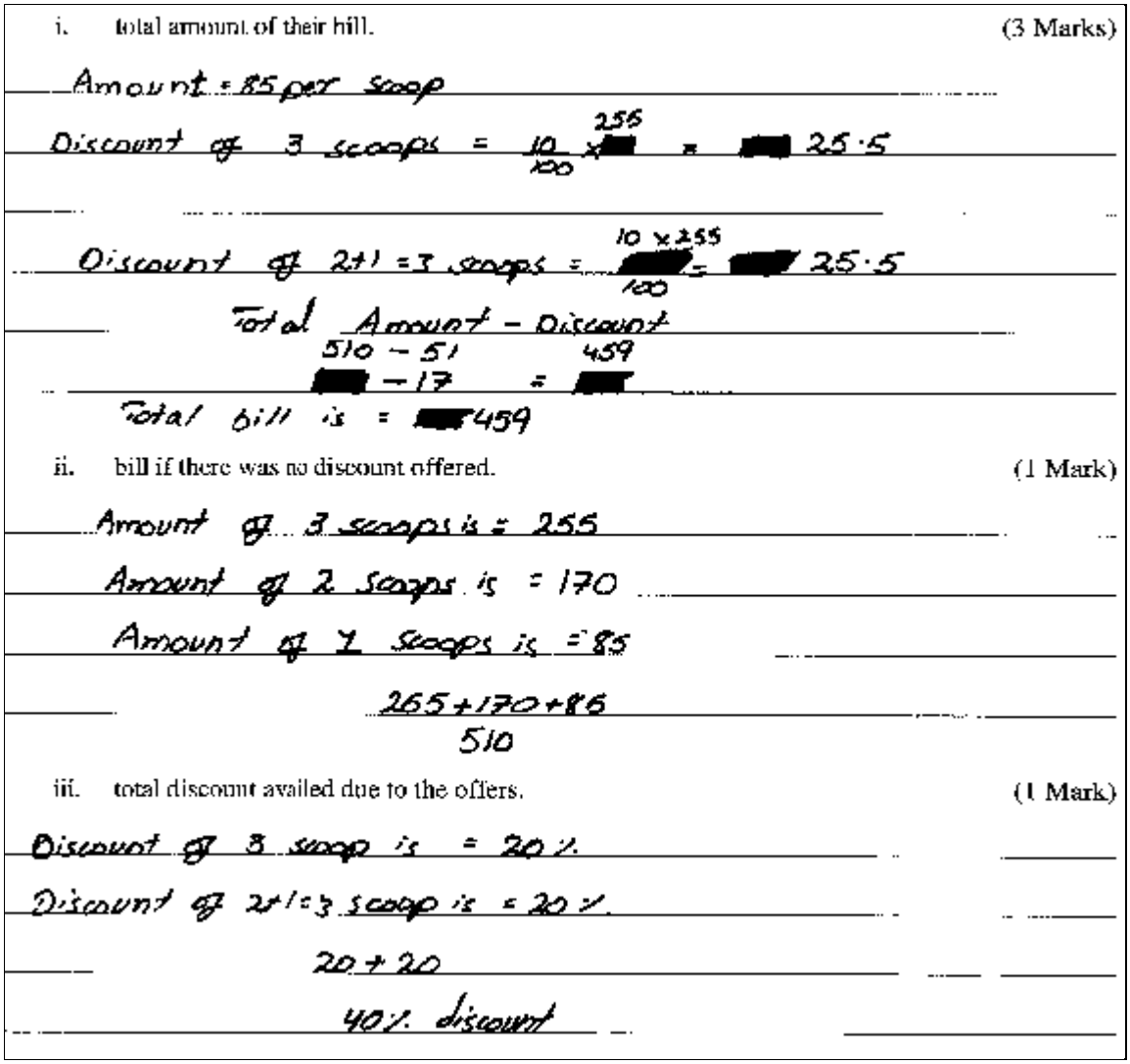
iii. total discount availed due to the offers. (1 Mark)

Total discount = 510 - 442

= Rs. 68

Weaker responses reflected that candidates used the percentages given in the offer, i.e. 10% and 20% to find the amount of bill in part i which is wrong because these percentages give discounts. It was evident from candidates' responses that they were not able to correctly translate the situation mathematically. Hence, a lot of guess work was also observed.

Example:



Question 4:

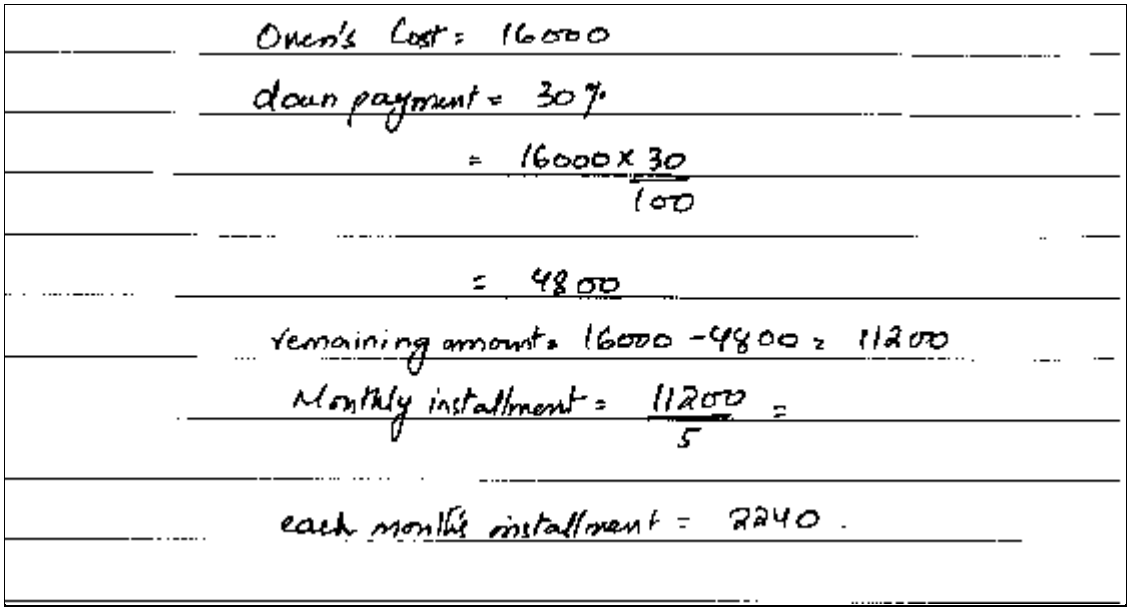
Candidates exhibited good performance in this question.

Question 4:

Zeeshan wants to buy a microwave oven that costs Rs 16,000. He would make a down payment of 30% followed by 5 equal monthly instalments. Calculate the instalment of each month.

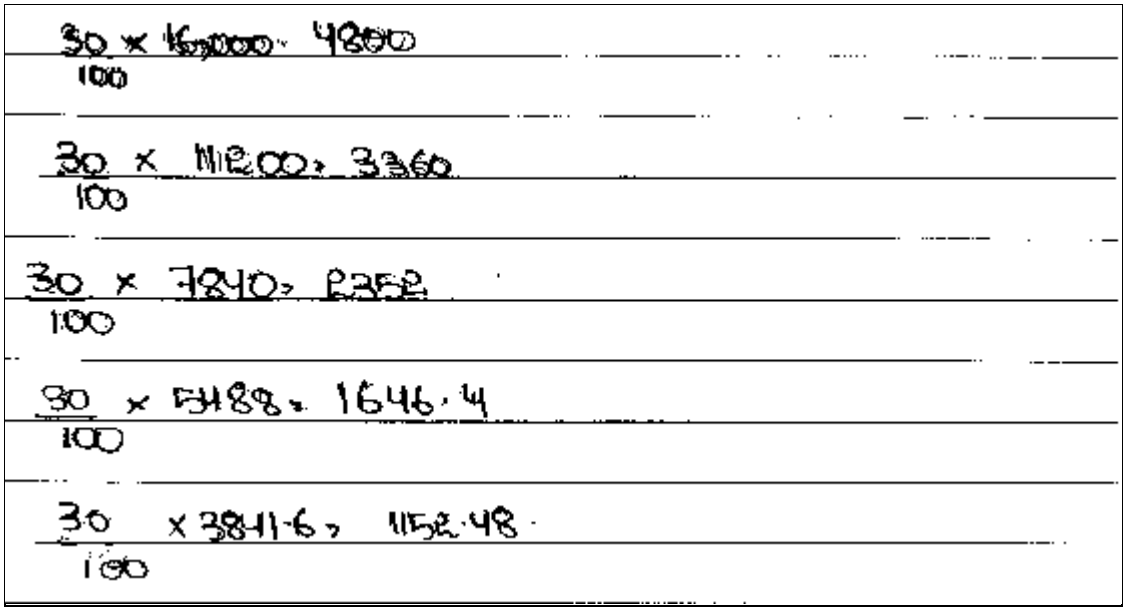
Better responses exhibited the understanding of down payment monthly instalments. Candidates used the given data to first find 30% down payment and then the 5 equal monthly instalments.

Example:



Weaker responses reflected that candidates mostly had difficulty in finding the monthly instalments. Candidates failed to translate the phrase 'followed by 5 equal monthly installments'. Some candidates calculated monthly installments before subtracting down payment while others did a lot of guess work after finding down payment.

Example:



Question 5:

Candidates showed average performance in this question.

Question 5:

i. Saad uses a telecommunication service to make an international call. The charge for the first 15 minutes is Rs 2.5 per minute. The charge for each additional minute is Rs 1. Calculate the cost if he makes a 1 hour-long call.

ii. For a certain week, the number of hours worked by Maaz each day is shown in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of Hours Worked	8	7	9	10	8	8

The rate of wage is Rs 215 per hour and is applicable as follows.

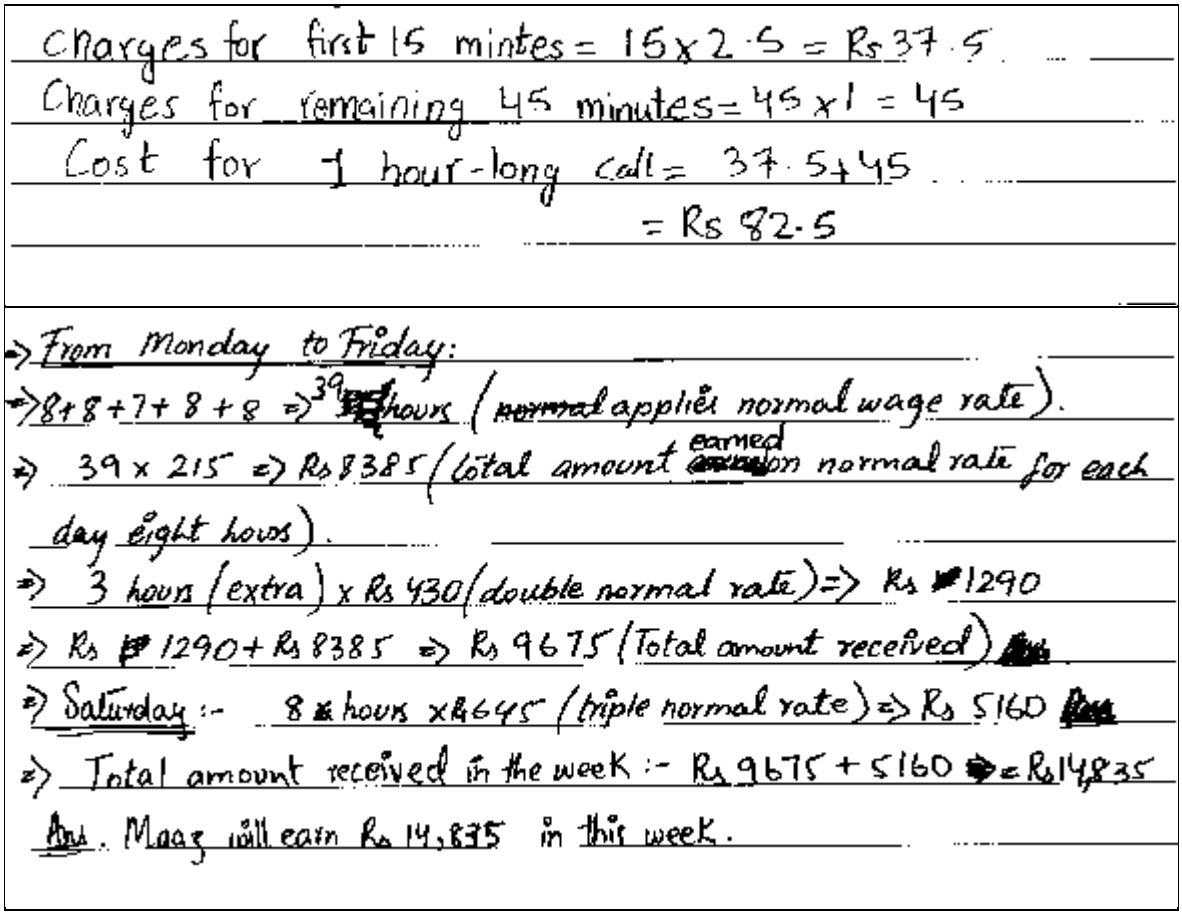
From Monday to Friday: the rate of wage applied will be doubled after 8 working hours.

On Saturday: the rate of wage will be three times for each working hour.

Calculate how much Maaz will earn at the end of this week.

Better responses of part i reflected that candidates were able to understand the situation and meaning of the phrases 'for each of the first 15 minutes' and 'each additional minute'. They used the given rates to find the cost of 1-hour long call. The same was observed in part ii, candidates used the conditions for weekdays and weekends on the given rate of wage to find the answer.

Example:



Weaker responses reflected that the candidates found working with the conditions given in part i and ii a little challenging. In part i, most of the candidates were able to find the cost of the first 15 minutes. After this, instead of finding the cost of additional 45 minutes, candidates found the cost of 60 additional minutes due to which their final answer was the cost of a call that is 1 hour 15 minutes long instead of the required 1 hour. In part ii, it was observed that candidates had difficulty handling these factors at one time:

i. The rate of wage within working hours in weekdays
ii. The rate of wage after working hours in weekdays
iii. The rate of wage for Saturday when there is no standard working hour duration
iv. The different number of hours worked each day by Maaz

Due to this there were many errors observed when candidates overlooked any of the above factors.

Example:

First 15 minutes charges = 2.5

If he makes a 1 hour long call his charges will be =

= 2.5 x 60

= 150 Costs.

Monday = 8 x 215 = Rs 1680.

Tuesday = 7x 215 = Rs 1505

Wednesday = 9 x 215(2) = Rs 3870

Thursday = 10 x 215(2) = Rs 4200

Friday = 8 x 215 = Rs 1680

Saturday = 8x 215(3) = Rs 5160

Total wage = Rs (1680 + 1505 + 3870 + 4200 + 1680 + 5160)

= Rs 18095

Maaz will paid Rs 18095 at the end of the week.

Question 6:

This question offered a choice between part a and b. Candidates chose to attempt part a more than part b. In part 6ai and 6bi, candidates showed average performance but in part 6aii and 6bii, candidates showed poor performance.

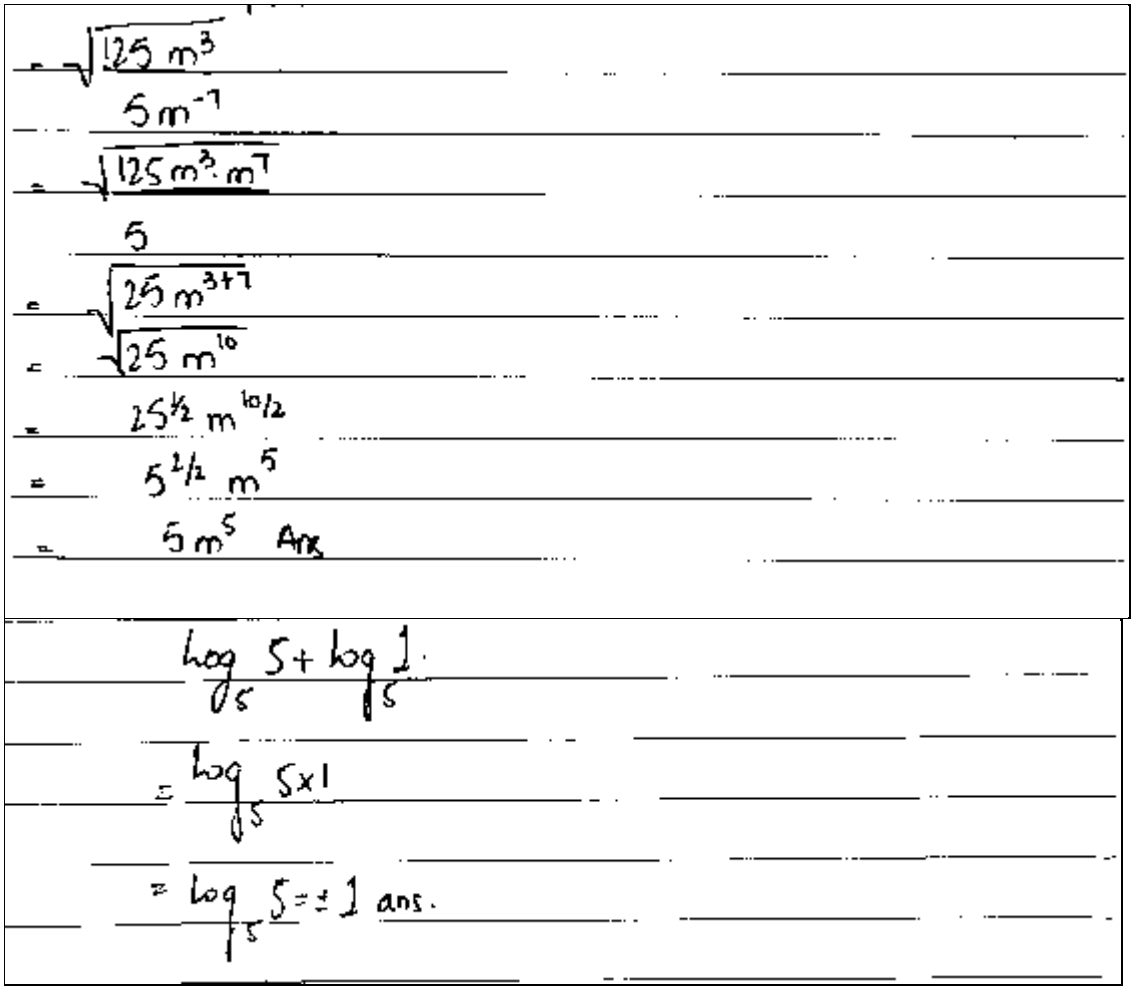
Question 6a:

i. Express $\sqrt{\frac{125m^3}{5m^{-7}}}$ in its simplest form.

ii. Simplify $\log_5 5 + \log_5 1$, giving your answer as an integer.

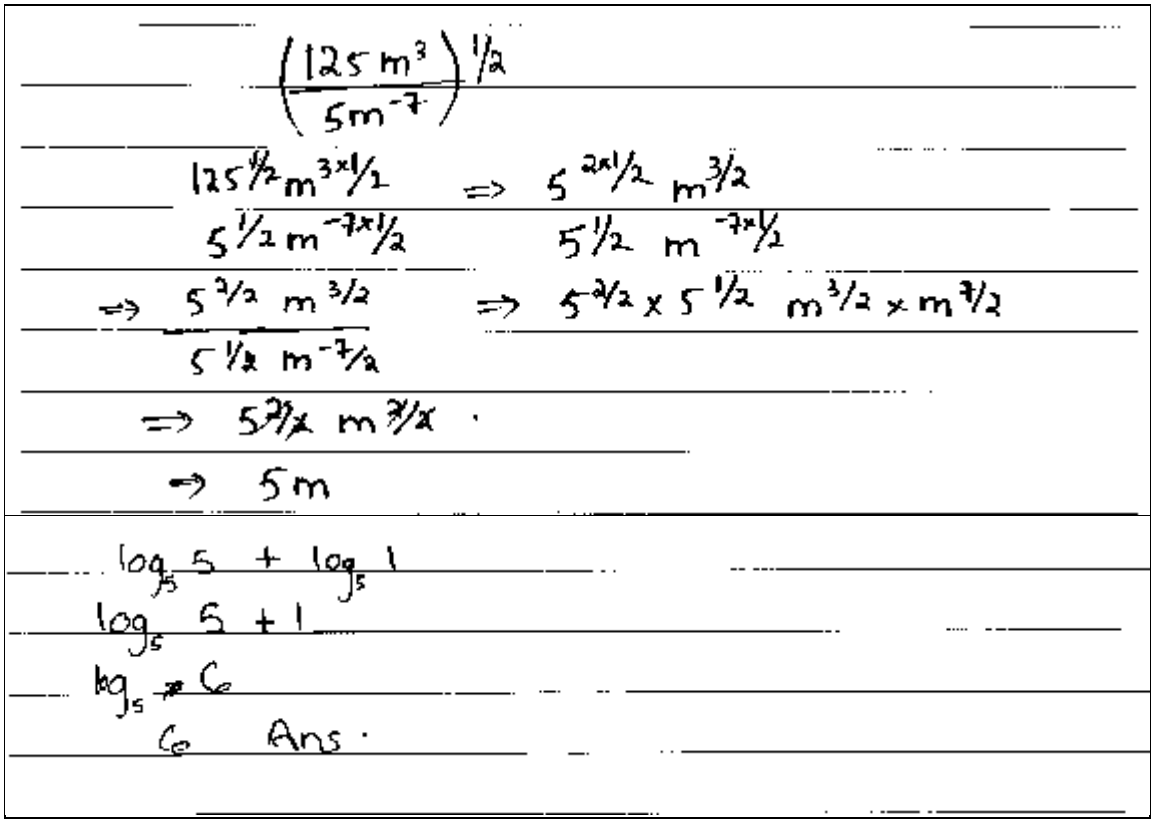
Better responses of part i exhibited the correct application of law of indices. Square root was expressed as the exponent 1/2, and after cancellation the answer was found. A few candidates were able to solve part ii using the correct rules of logarithms and finding answer as an integer.

Example:



Weaker responses of part i reflected misconceptions about the use of all the laws of indices. There was a great confusion about when to add the indices and when to multiply. Candidates had difficulty dealing with the square root. Many candidates carelessly did not change the sign when moving *m* from denominator to numerator. In part ii, candidates were able to express the given as a single term and then put $\log_5 5 = 1$.

Example:



Question 6b:

i. Without using a calculator, show that $\frac{1}{\sqrt{128}}$ is equal to $2^{-\frac{7}{2}}$.

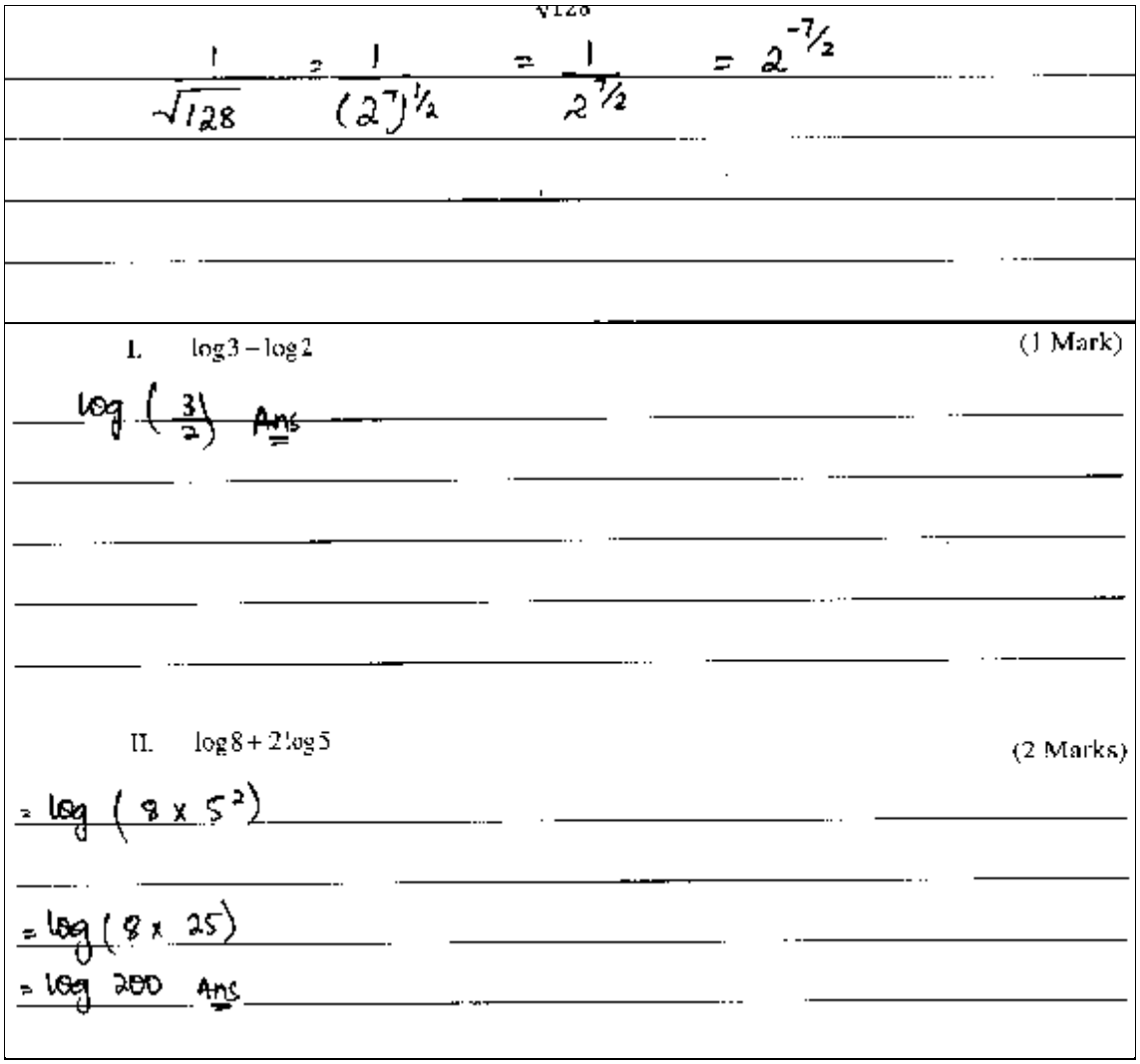
ii. Simplify and express each of the following as a single logarithmic term.

I. $\log 3 - \log 2$

II. $\log 8 + 2\log 5$

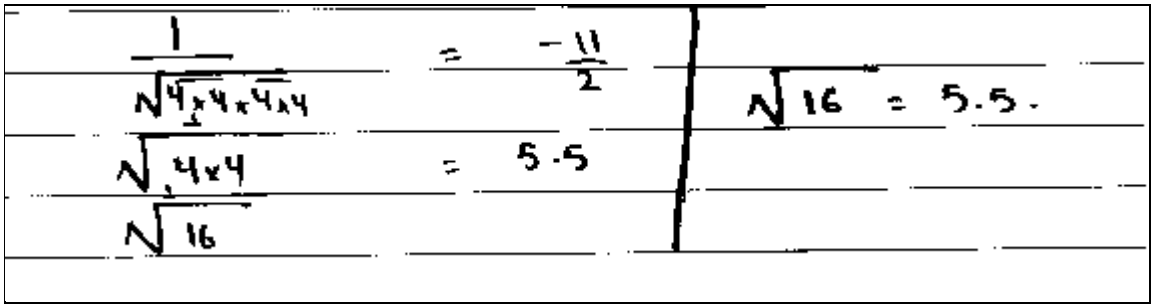
Better responses reflected that in part i, candidates were able to express 128 as a power of 2. Laws of indices were used correctly to find the answer. Part ii was done correctly by few candidates who followed the instructions to express answer as a 'single logarithmic term'.

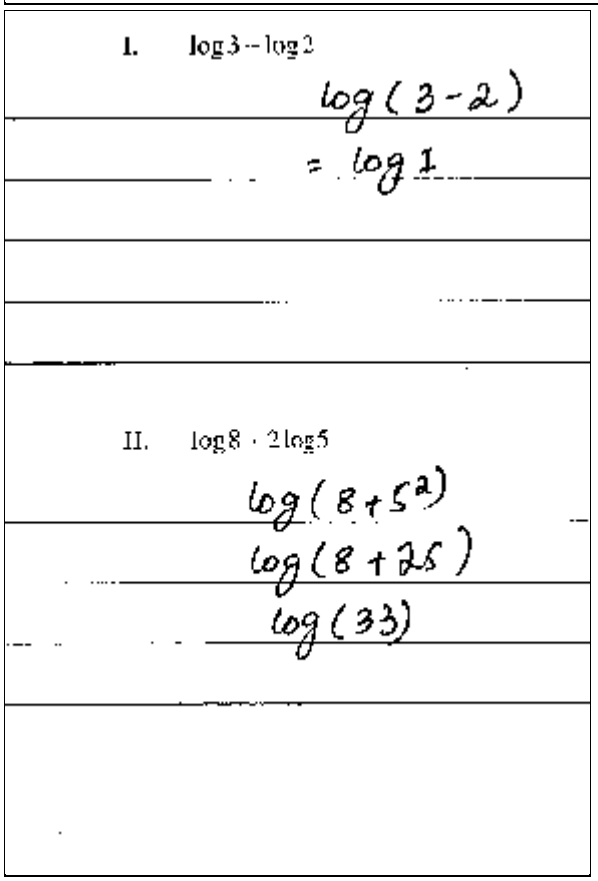
Example:



Weaker responses in part i reflected that candidates had difficulty in expressing 128 as a power of 2. Since the answer was given in the question, candidates used all kinds of incorrect short cuts to get there. In part ii, it was observed that candidates were able to apply law of logarithm on addition in part II more easily as compared law of logarithm on subtraction in part I. The most frequent incorrect solution was log 3 – log 2 = log (3 –2) = log 1.

Example:





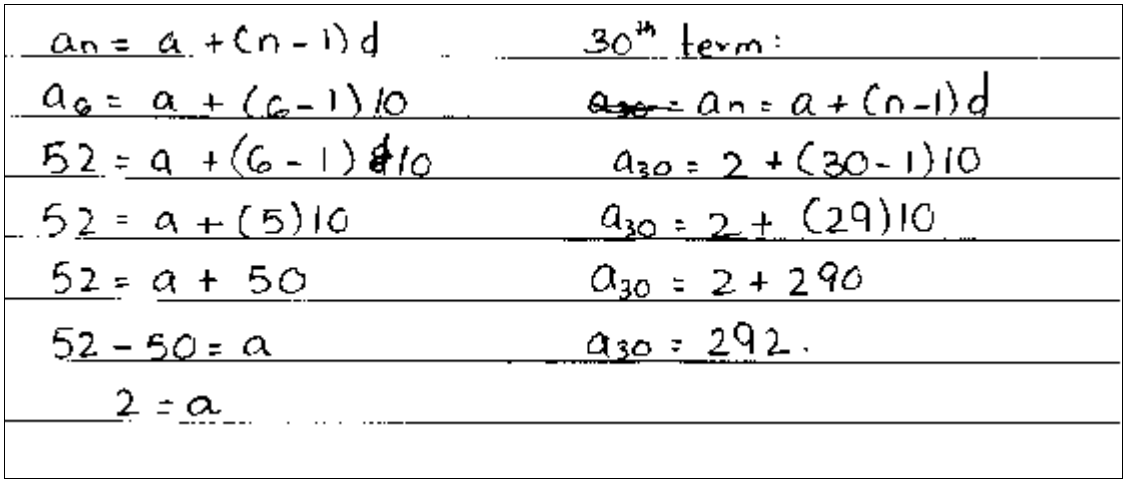
Question 7:

An arithmetic sequence has a common difference of 10 and its 6^{th} term is 52. Find its 30^{th} term.

This was a well attempted question.

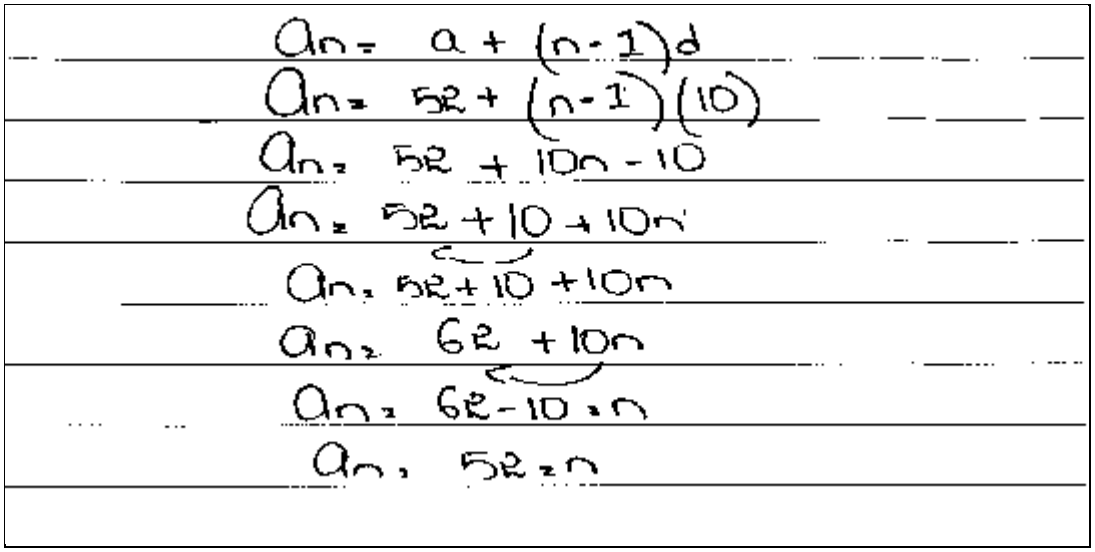
Better responses exhibited that most of the candidates had knowledge of general term of arithmetic sequence. They used sixth term to find the value of *d*. This value of *d* was used to find the 30^{th} term of the sequence.

Example:



Weaker responses represented that candidates were not able to use the given values in the question to find the value of *d*. They used incorrect general form of arithmetic sequence, arithmetic series and geometric sequence. Many candidates did not understand that first the common difference needs to be found using the given data before finding the 30^{th} term.

Example:



Question 8:

This question offered a choice between part a and b. A vast majority of candidates chose part b which was also attempted better than part a.

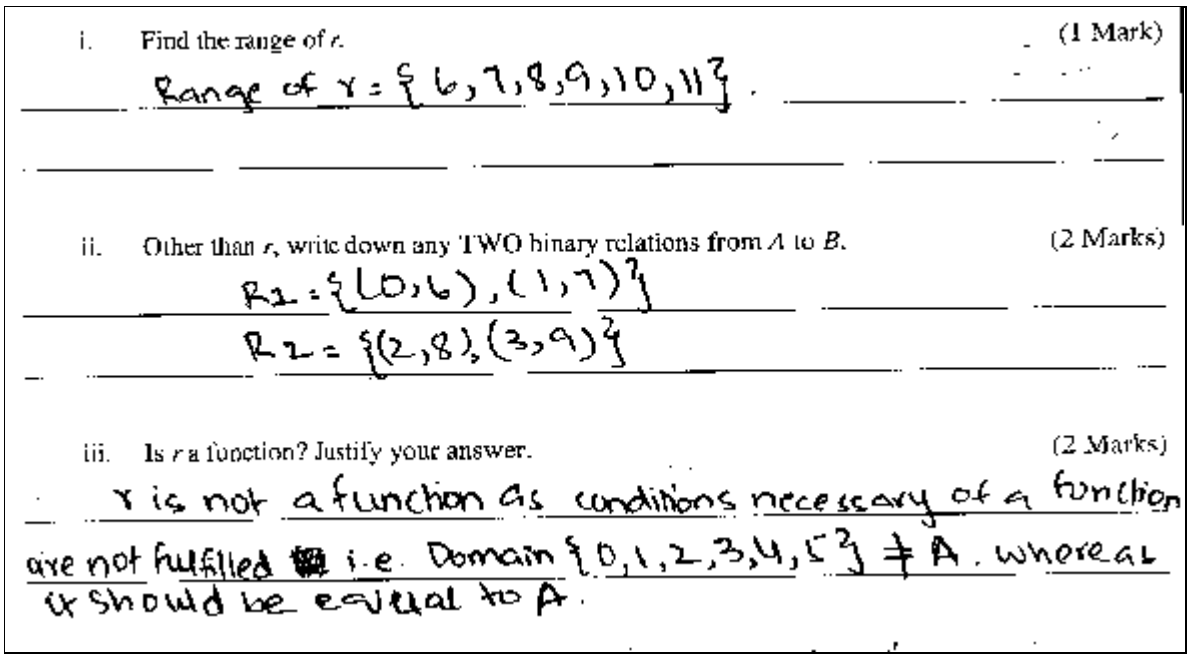
Question 8a:

A binary relation from $A = \{0, 1, 2, 3, 4, 5, 6\}$ to $B = \{6, 7, 8, 9, 10, 11, 12\}$ is defined as $r = \{(0, 6), (1, 7), (2, 8), (3, 9), (4, 10), (5, 11)\}$.

i. Find the range of r.
ii. Other than r, write down any TWO binary relations from A to B.
iii. Is r a function? Justify your answer.

Better responses reflected good understanding of binary relations, its domain and range and function. Candidates formed a variety of binary relations from A to B . In part iii, r was not a function which only few candidates were able to identify. The correct reason was also stated.

Example:



i. Find the range of r. (1 Mark)

Range of $r = \{6, 7, 8, 9, 10, 11\}$.

ii. Other than r, write down any TWO binary relations from A to B. (2 Marks)

$R_1 = \{(0, 6), (1, 7)\}$

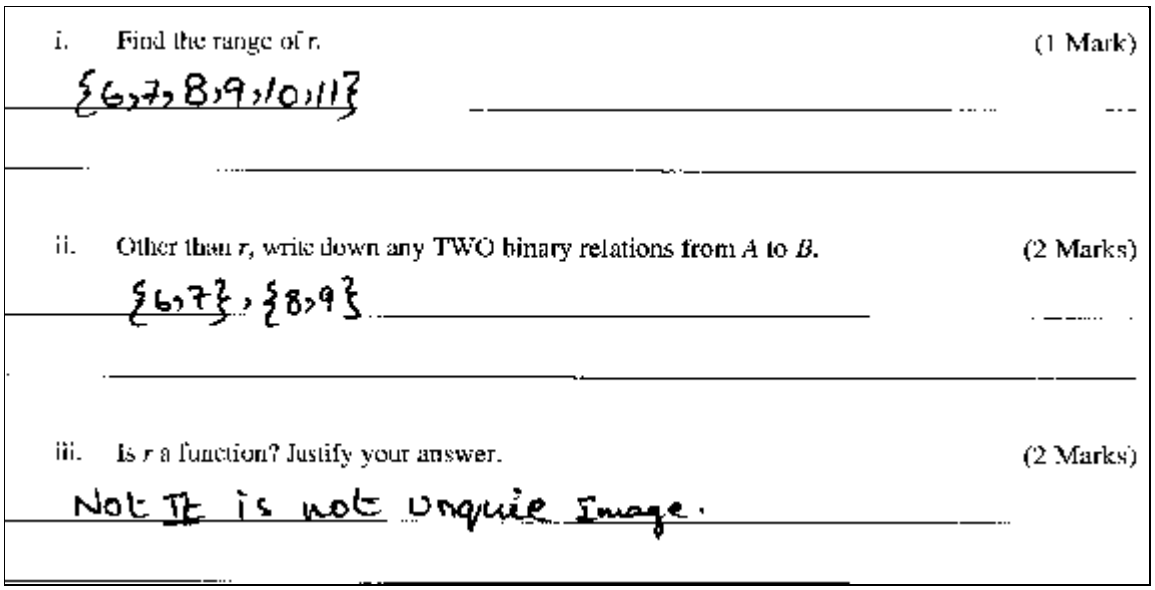
$R_2 = \{(2, 8), (3, 9)\}$

iii. Is r a function? Justify your answer. (2 Marks)

r is not a function as conditions necessary of a function are not fulfilled i.e. Domain $\{0, 1, 2, 3, 4, 5\} \neq A$. whereas it should be equal to A.

Weaker responses reflected that most of the candidates had understanding of binary relation and its domain and range but they were unclear about the conditions that should be fulfilled for a binary relation to be a function. Many candidates stated that the given *r* is a function with incorrect reasons. Due to the pattern of r, they assumed it as one-one function and overlooked the fact that the domain of *A* is not equal to the domain of *r*.

Example:



i. Find the range of r. (1 Mark)

{6,7,8,9,10,11}

ii. Other than *r*, write down any TWO binary relations from *A* to *B*. (2 Marks)

{6,7}, {8,9}

iii. Is *r* a function? Justify your answer. (2 Marks)

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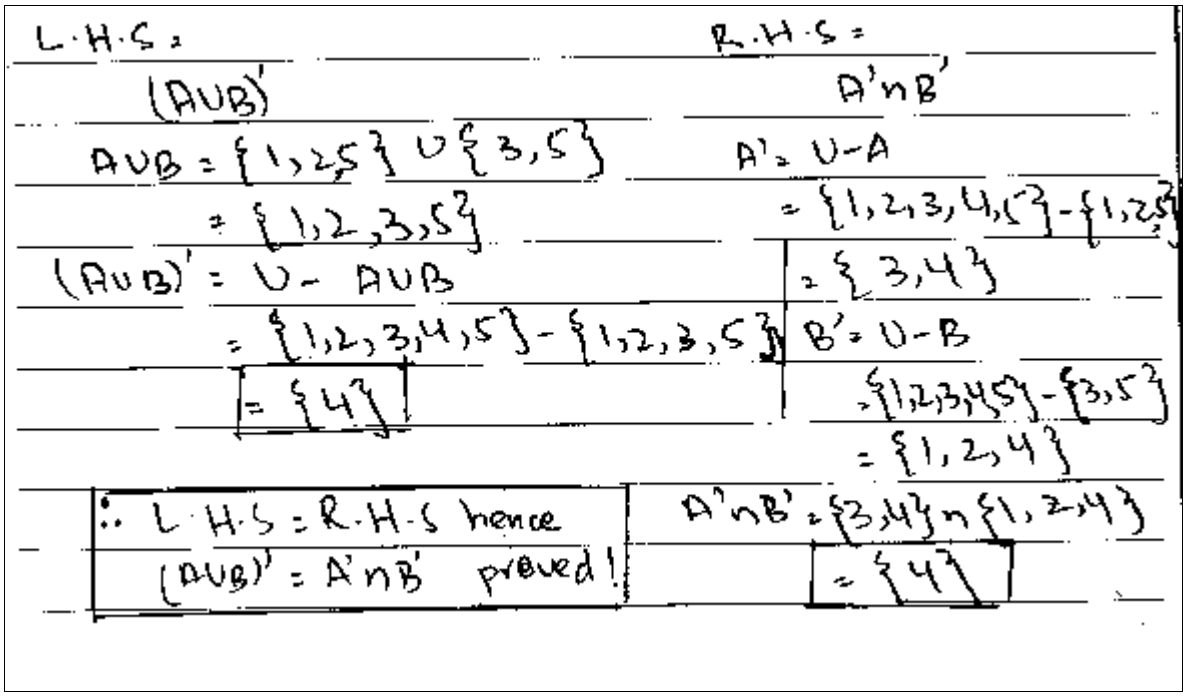
Question 8b:

If $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 5\}$ and $B = \{3, 5\}$, then show that $(A \cup B)' = A' \cap B'$.

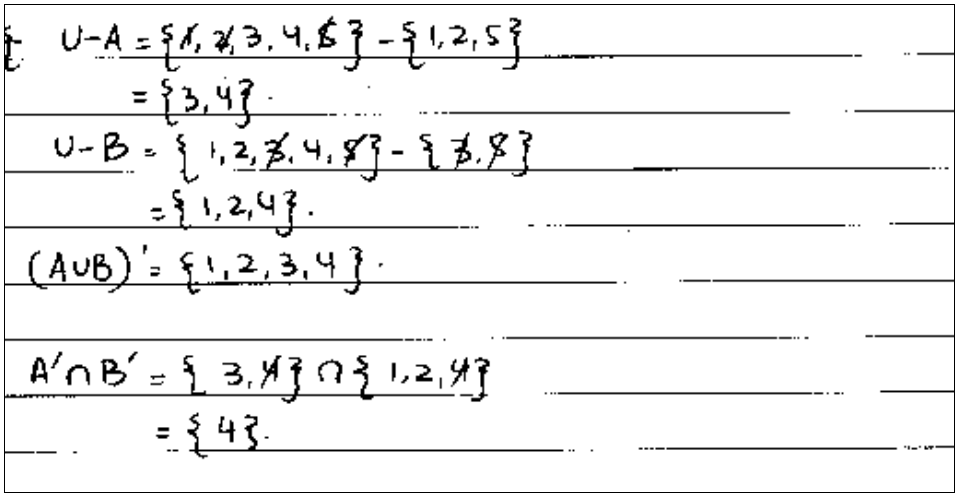
Better responses showed that candidates had understanding of operations on sets. The left hand side and right hand side of the equation was solved separately, both came out as { 4 }.

Example:



Weaker responses displayed that candidates made careless errors while solving the two sides $(A \cup B)'$ and $A' \cap B'$. Union was often replaced with intersection. Errors in finding difference were common as well.

Example:



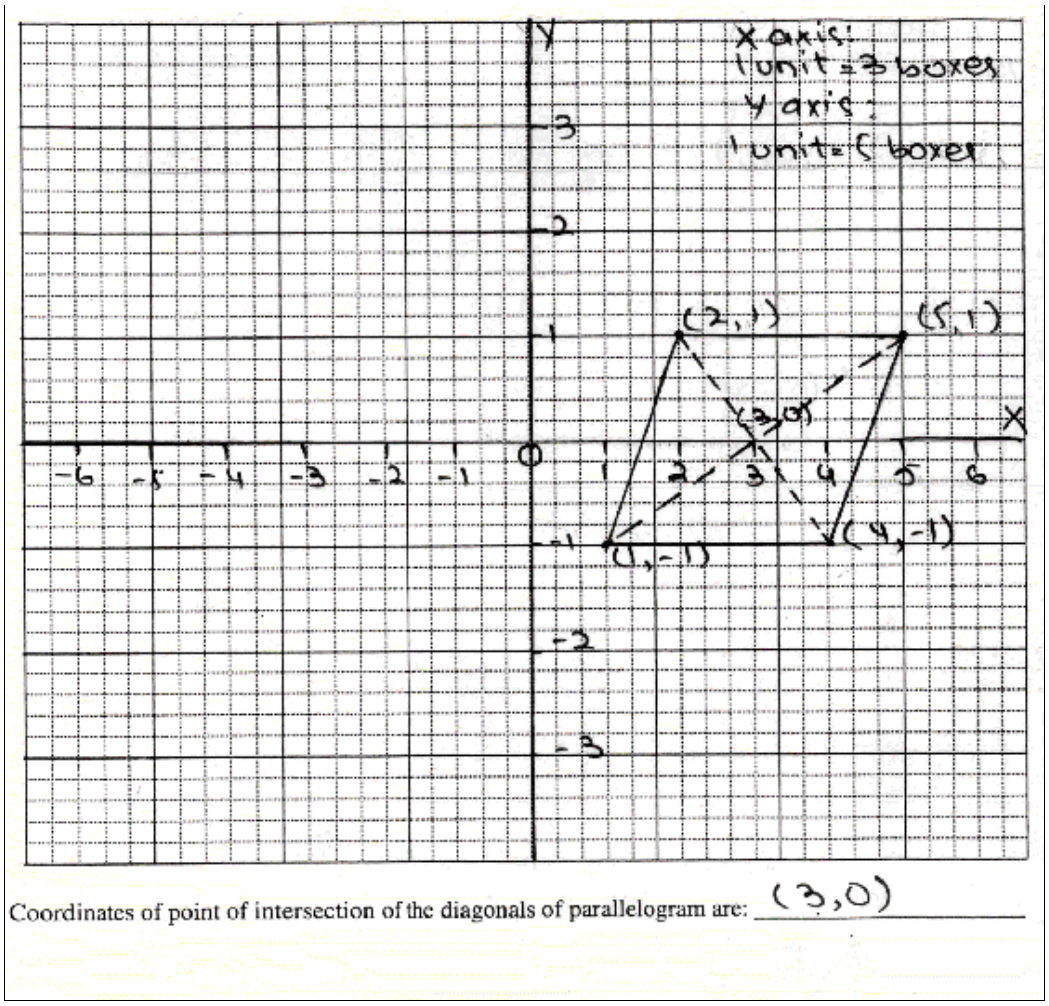
Question 9:

Show graphically that the points (1, – 1), (2, 1), (4, – 1) and (5, 1) are the vertices of a parallelogram. Also, find the coordinates of point of intersection of the diagonals.

This was a well attempted question.

Better responses plotted the given four vertices of parallelogram correctly on the given graph. The *x*-axis and *y*-axis were made with proper scaling too. The diagonals were drawn by joining the opposite vertices and the coordinates of diagonal were found.

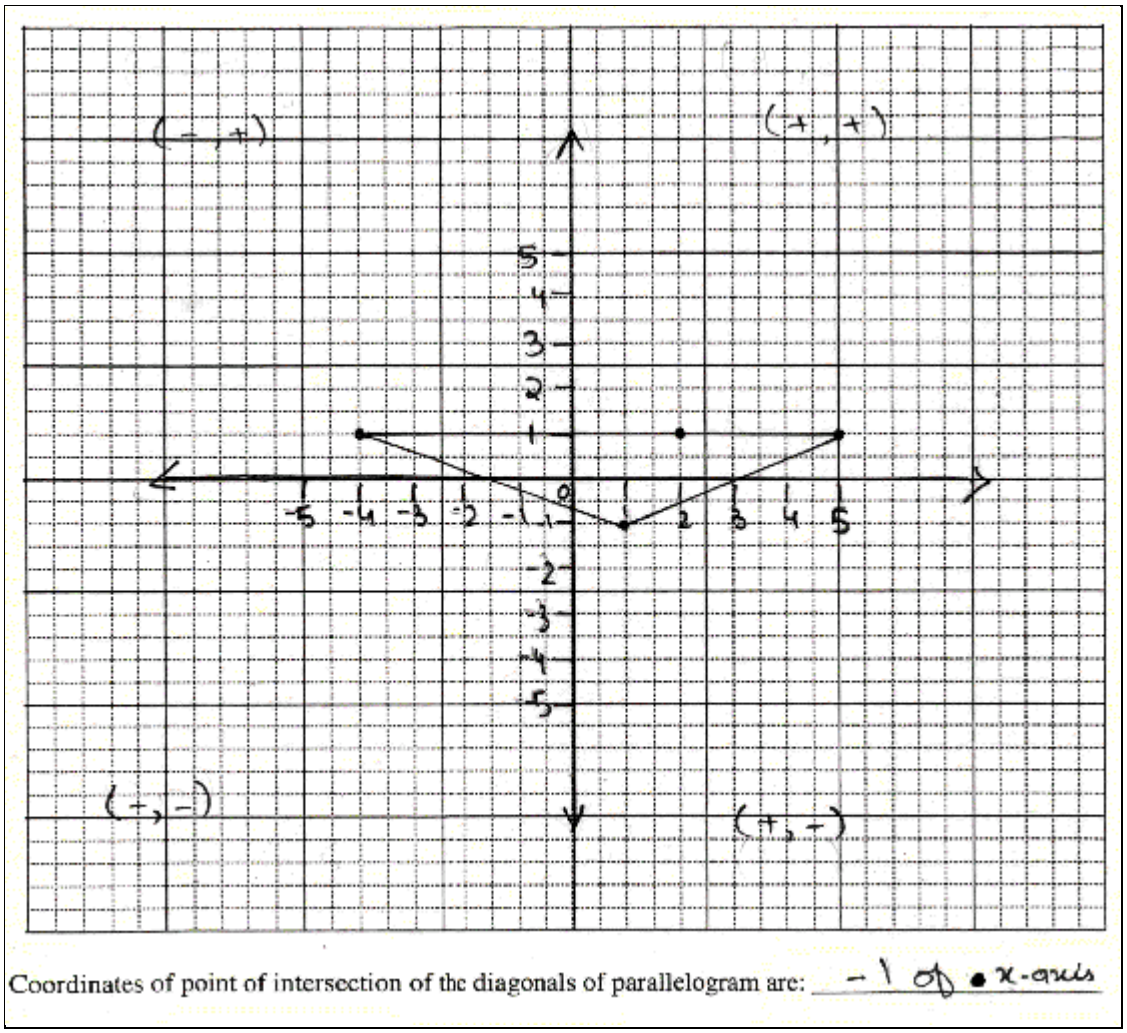
Example:



Coordinates of point of intersection of the diagonals of parallelogram are: (3, 0)

Weaker responses reflected weak skills of plotting on a graph paper. The positive part of *x*-axis was left hand side instead of right hand side. The same was observed for *y*-axis. The given ordered pairs were plotted incorrectly because candidates took the order par as (*y*, *x*) instead of the standard (*x*, *y*). Many candidates lost the last mark because they did not know how to draw the diagonal of a parallelogram.

Example:



Question 10:

This question offered a choice between part a and b. Candidates chose to attempt part a more than part b. Both parts were well-attempted.

Question 10a:

i. A small office has a staff of 20 employees. The distance (in kilometres) of the office from their homes is shown in the given table. Complete the table and find arithmetic mean for this data.

Distance (km)	**Number of Employees**		
$1-4$	2		
$5-8$	1		
$9-12$	6		
$13-16$	10		
$17-20$	1		
Total	$\sum f = 20$	–	

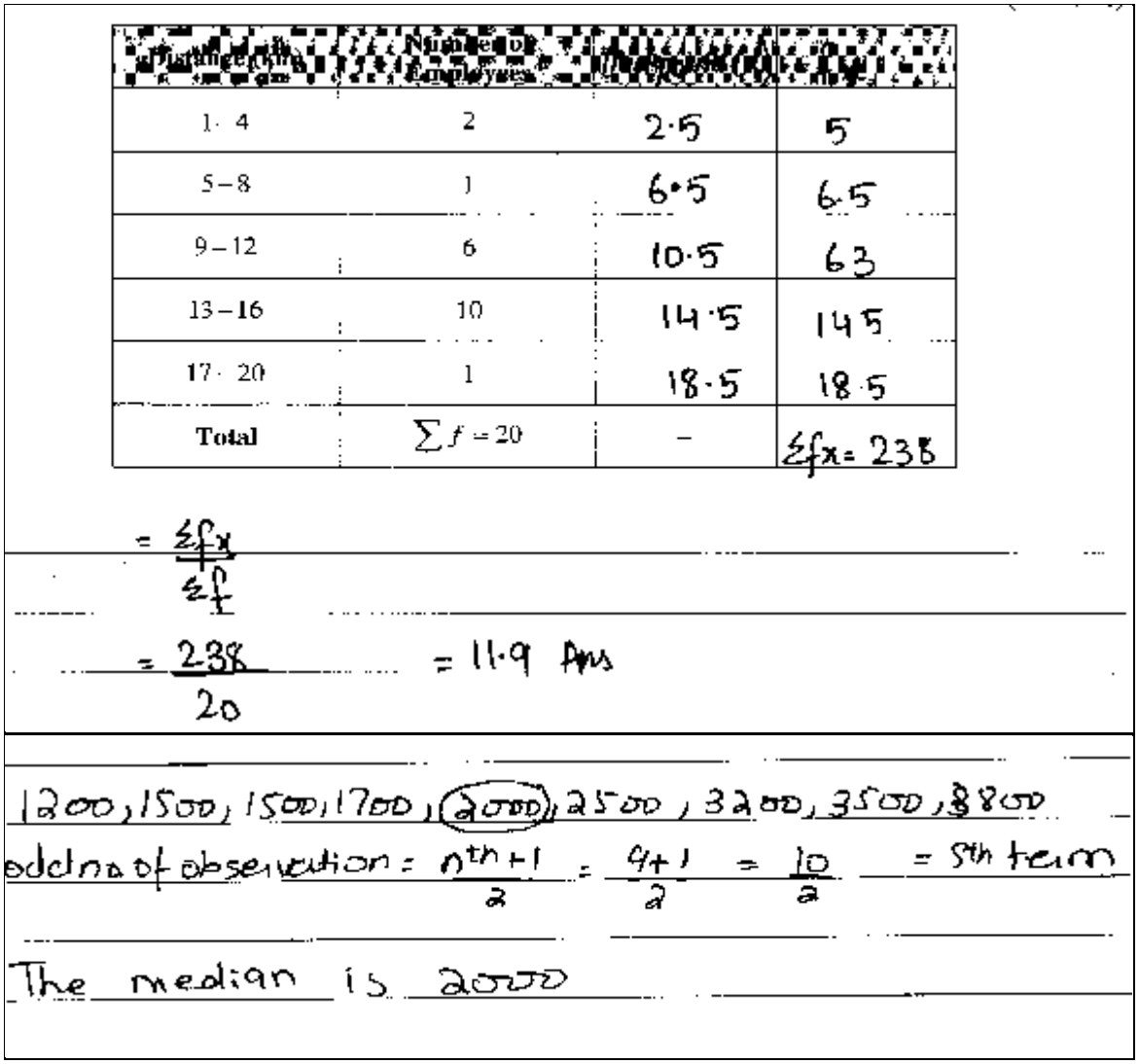
ii. On a certain day, 9 friends spend the following amount (in Rupees) on a shopping trip.

3800, 2500, 1200, 1500, 1700, 2000, 1500, 3200, 3500

Find the median amount that was spent.

Better responses in part i displayed clear concepts of method of finding mean from grouped data. The frequency and class marks were found. The formula for grouped mean was used to find the answer. In part ii, the data was first arranged in ascending or descending order. The value of *n* is 9 so the middle value or median was found.

Example:



Number of Employees			
1 - 4	2	2·5	5
5 – 8	1	6·5	6.5
9 – 12	6	10·5	63
13 – 16	10	14·5	145
17 - 20	1	18·5	18·5
Total	$\sum f = 20$	–	$\sum fx = 238$

$= \frac{\sum fx}{\sum f}$

$= \frac{238}{20}$ = 11·9 Ans

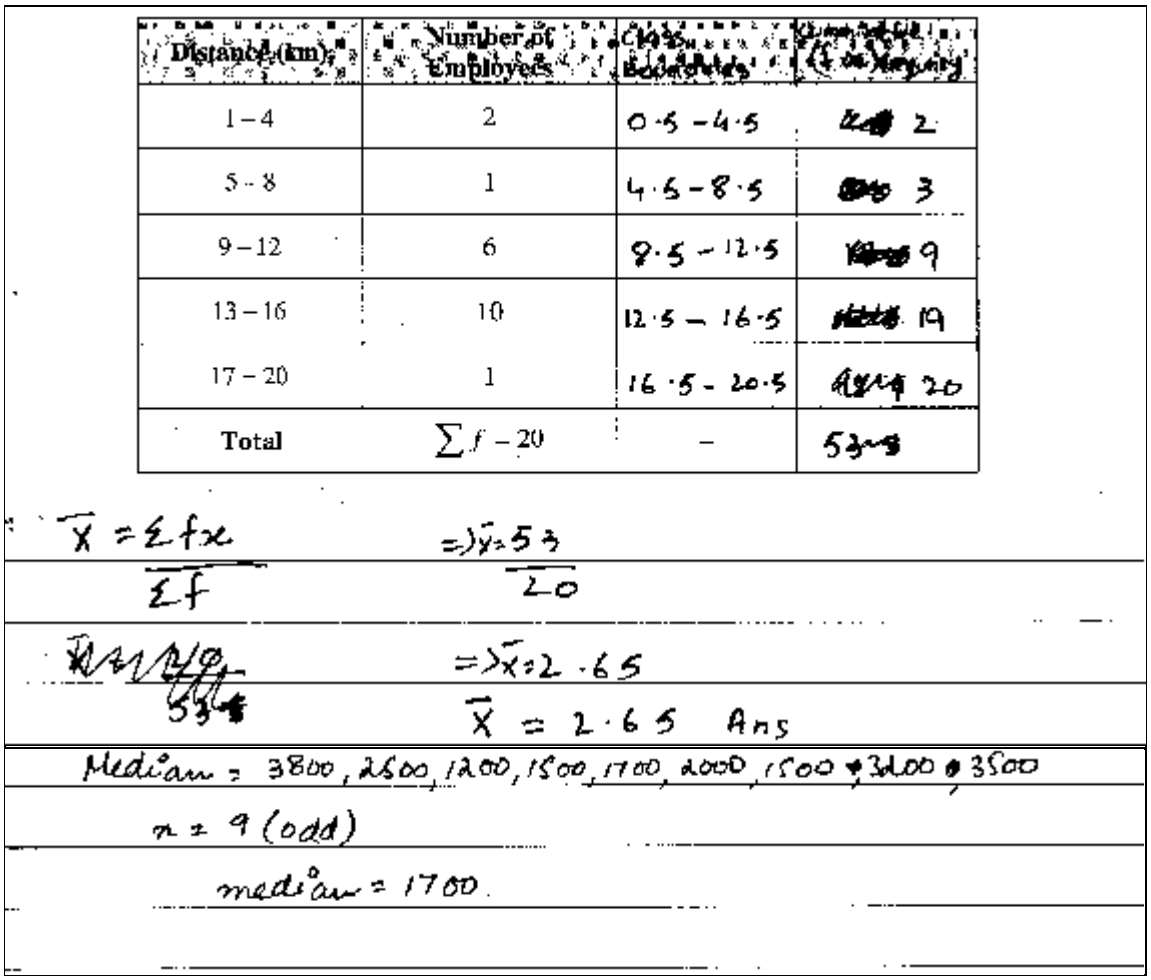
1200, 1500, 1500, 1700, (2000), 2500, 3200, 3500, 8800

odd no of observation : $\frac{n^{th}+1}{2} = \frac{9+1}{2} = \frac{10}{2}$ = 5th term

The median is 2000

Weaker responses of part i reflected that candidates could not identify what is to be found in the blank columns. Some candidates found class boundaries while others tried but failed to find mid-point. Minor errors were also common which led to incorrect answers. For part ii, the most frequent error was that candidates did not arrange the data into ascending or descending order before finding the median hence the most frequent incorrect answer was 1700. Due to this mistake a large number of candidates scored only 1 mark.

Example:



Distance (km)	Number of Employees	Class Boundaries	(f × x)
1 – 4	2	0·5 – 4·5	2
5 – 8	1	4·5 – 8·5	3
9 – 12	6	8·5 – 12·5	9
13 – 16	10	12·5 – 16·5	19
17 – 20	1	16·5 – 20·5	20
Total	$\sum f = 20$	–	53

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ $\Rightarrow \bar{x} = \frac{53}{20}$

$\Rightarrow \bar{x} = 2.65$

$\bar{x} = 2.65$ Ans

Median = 3800, 2500, 1200, 1500, 1700, 2000, 1500, 3200, 3500

$n = 9$ (odd)

median = 1700.

Question 10b:

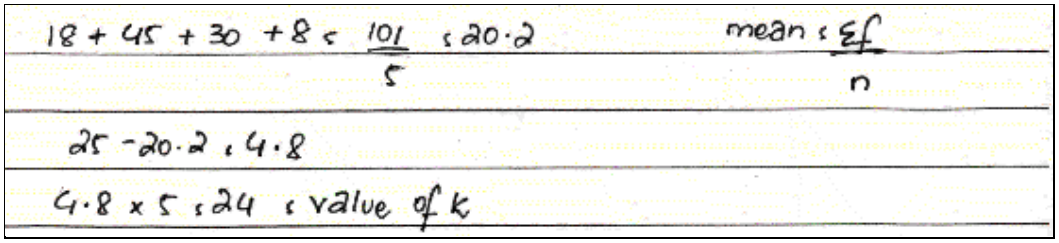
i. If the mean of 5 numbers 18, *k*, 45, 30 and 8 is 25, then find the value of *k*.

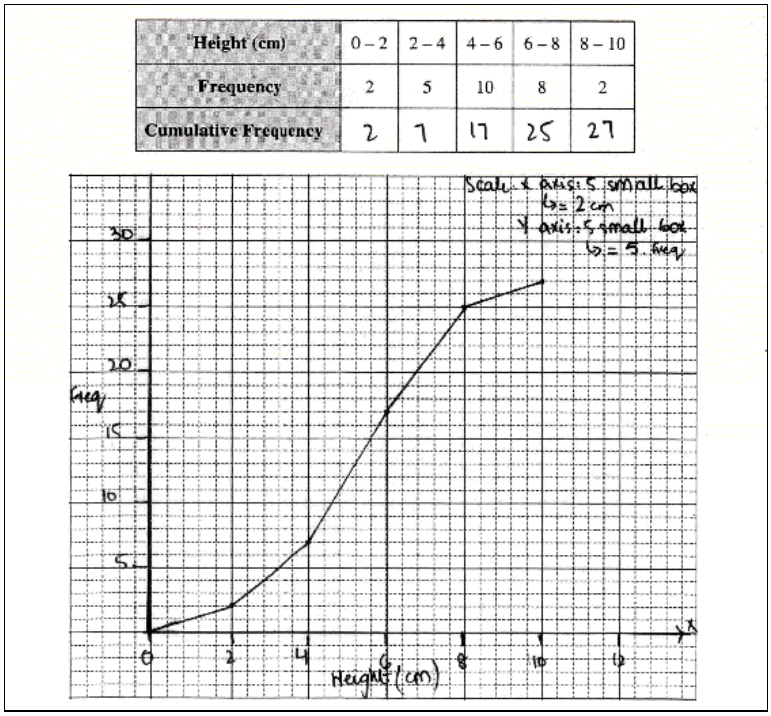
ii. The given data shows the height (in centimetres) of 27 plants. Complete the given table and use it to construct a cumulative frequency curve.

Height (cm)	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10
Frequency	2	5	10	8	2
Cumulative Frequency					

Better responses of part i reflected clear understanding of arithmetic mean of ungrouped data. The given numbers were added to find 101 + *k* which was substituted in the formula to get the value of *k*. Part ii displayed correct drawing of cumulative frequency curve. Candidates took class intervals on *x*-axis and cumulative frequency on *y*-axis to plot the cumulative frequency that they found from the given data.

Example:





Weaker responses of part i showed either error in arithmetic calculations or incorrect substitution in formula. Part ii reflected the misconception among candidates that class boundaries were required to draw the cumulative frequency curve (Since the class intervals are continuous, class boundaries are not required). There were errors in scaling of graph such as taking cumulative frequency on *x*-axis and class intervals on *y*-axis. Another common mistake was that after plotting the point and drawing the curve it was not traced back to 0 on *x*-axis. Many candidates plotted frequency curve, frequency polygon and cumulative frequency polygon instead of the required cumulative frequency curve

Example:

$\frac{18+45+30+8}{5} = \frac{101}{5}$

$k = 20.2$

Height (cm)	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10
Frequency	2	5	10	8	2
Cumulative Frequency	1	3	5	7	9