AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XI

ANNUAL EXAMINATIONS (THEORY) 2023

Mathematics Paper II

Time: 1 hour and 30 minutes   Marks: 50

INSTRUCTIONS

Please read the following instructions carefully.

1. Check your name and school information. Sign if it is accurate.

I agree that this is my name and school.
Candidate's Signature

RUBRIC

2. There are EIGHT questions. Answer ALL questions. Choices are specified inside the paper.

3. When answering the questions:

   Read each question carefully.
   Use a black pointer to write your answers. DO NOT write your answers in pencil.
   Use a black pencil for diagrams. DO NOT use coloured pencils.
   DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
   Complete your answer in the allocated space only. DO NOT write outside the answer box.

4. The marks for the questions are shown in brackets ( ).

5. A formulae list is provided on page 2 and 3. You may refer to it during the paper, if you wish.

6. You may use a scientific calculator if you wish.

List of Formulae

Note:

- All symbols used in the formulae have their usual meaning.

Complex Numbers

$|z| = \sqrt{a^2 + b^2}$

Matrices and Determinants

$A_{ij} = (-1)^{i+j} M_{ij}$ $\quad$ $AdjA = (A_{ij})^t$ $\quad$ $A^{-1} = \frac{1}{|A|} AdjA$

Sequence & Series and Miscellaneous Series

$a_n = a_1 + (n-1)d$ $\quad$ $A = \frac{a+b}{2}$ $\quad$ $S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$

$a_n = a_1 r^{n-1}$ $\quad$ $G = \pm\sqrt{ab}$ $\quad$ $H = \frac{2ab}{a+b}$

$S_n = \frac{a_1(1-r^n)}{1-r}$ if $|r| < 1$ $\quad$ $S_n = \frac{a_1(r^n - 1)}{r-1}$ if $|r| > 1$ $\quad$ $S_\infty = \frac{a_1}{1-r}$, where $|r| < 1$

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\quad$ $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad$ $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$

Permutations, Combinations and Probability

${}^nP_r = \frac{n!}{(n-r)!}$ $\quad$ ${}^nC_r = \frac{n!}{(n-r)!r!}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\quad$ $P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\quad$ $P(A \cap B) = P(A) \times P(B)$

Binomial Theorem and Mathematical Induction

$(a+x)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \binom{n}{3}a^{n-3}x^3 + \ldots + \binom{n}{n-1}a^1x^{n-1} + x^n$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots$

$T_{r+1} = \binom{n}{r}a^{n-r}x^r$

Quadratic Equation

$x^2 - Sx + P = 0$ $\quad$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad$ $D = b^2 - 4ac$

Introduction to Trigonometry and Trigonometric Identities

$l = r\theta$ $\quad \sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ $\quad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\quad \tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ $\quad \sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ $\quad \tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$ $\quad a^2 = b^2 + c^2 - 2bc\cos\alpha$ $\quad \dfrac{a-b}{a+b} = \dfrac{\tan\dfrac{\alpha-\beta}{2}}{\tan\dfrac{\alpha+\beta}{2}}$

$\cos P - \cos Q = -2\sin\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$ $\quad \sin P - \sin Q = 2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\cos P + \cos Q = 2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$ $\quad \sin P + \sin Q = 2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$ $\quad \tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

$\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$

Application of Trigonometry

$\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}ac\sin\beta = \dfrac{1}{2}ab\sin\gamma$ $\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \dfrac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$ $\quad R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma}$

$r_1 = \dfrac{\Delta}{s-a}$, $r_2 = \dfrac{\Delta}{s-b}$ and $r_3 = \dfrac{\Delta}{s-c}$ $\quad r = \dfrac{\Delta}{s}$ $\quad R = \dfrac{abc}{4\Delta}$

Graphs of Trigonometric Functions, Inverse Trigonometric Functions and Solution of Trigonometric Equations

$\sin^{-1}A \pm \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} \pm B\sqrt{1-A^2}\right)$ $\quad \cos^{-1}A \pm \cos^{-1}B = \cos^{-1}\left(AB \mp \sqrt{(1-A^2)(1-B^2)}\right)$

$\tan^{-1}A \pm \tan^{-1}B = \tan^{-1}\left(\dfrac{A \pm B}{1 \mp AB}\right)$

Q.1. (Total 4 Marks)

For the polynomial $ax^3 - 3x^2 + 9x - 27$, the factors are $(x-3)$ and (x^2+9).

i. Find the value of a.

ii. Hence, factorise (x^2+9).

Q.2. (Total 6 Marks)

i. If the determinant of the matrix $\begin{bmatrix} 0 & -1 & 1 \\ 1 & a & 2 \\ 1 & 2 & 1 \end{bmatrix}$ is 0, then find the value of a. (3 Marks)

ii. A matrix is defined as $M = \begin{bmatrix} 0 & 2 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}$. Show that $|2M| = -8$. (3 Marks)

Q.3. (Total 6 Marks)

i. The 1st, 2nd and 3rd terms of an arithmetic sequence are 7, 12 and 17 respectively.

Find the 10th term. (2 Marks)

ii. It is given that in a geometric sequence, the ratio of 2nd and 4th term is 1:9.

Find the common ratio. (2 Marks)

iii. The sum of n terms of a series is given by $S_n = \sum_{k=1}^{n}(k^2 - 1)$.

Show that $S_n = \frac{n}{6}\left(2n^2 + 3n - 5\right)$. (2 Marks)

Q.4. (Total 6 Marks)

i. On a sports day, 32 students compete in a race. The top three runners receive gold, silver, and bronze medals respectively. Find the number of possibilities for attaining the top three positions if

I. all students are present on sports day. (2 Marks)

II. two students are absent on sports day. (1 Mark)

ii. There were 47 people at an annual dinner of a company.

I. Each of them shook hands with everyone else. Find the total number of handshakes. (2 Marks)

II. If a wall clock given as a gift randomly to 5 different people at the dinner, then find the number of possibilities for winning the gifts. (1 Mark)

Q.5. (Total 6 Marks)

i. Using mathematical induction, prove that for $n \geq 1$, $1 + 4 + 7 + (3n - 2) = \frac{n(3n-1)}{2}$. (4 Marks)

ii. Show that the series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + ...$ is a convergent geometric series. (2 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.6.)

Q.6. (Total 6 Marks)

a. The relationship between the roots of a quadratic equation are given such that $3\alpha - \beta = 7$ and $\alpha = \frac{\beta}{2} + 1$, where α and β are the roots of a quadratic equation. Find the

i. values of α and β. (3 Marks)

ii. values of $\alpha + \beta$ and $\alpha\beta$. (2 Marks)

iii. quadratic equation. (1 Mark)

(ATTEMPT EITHER PART a OR PART b OF Q.6.)

b. (Total 6 Marks)

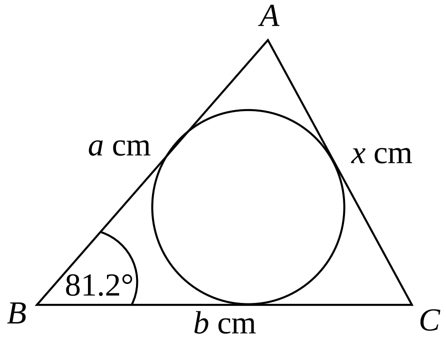
i. Find the cube roots of -64. (3 Marks)

ii. If α and β are the roots of the quadratic equation $x^2+3x+5=0$, then find the quadratic equation whose roots are α^2 and β^2. (3 Marks)

(ATTEMPT EITHER ANY TWO OF a, b AND c FOR Q.7.)

Q.7. (Total 10 Marks)

a. Consider the given diagram. (5 Marks)



- $(a+b)\text{ cm} = 27\text{ cm}$.
- Area $= \Delta = 84\text{ cm}^2$.
- Radius of inscribed circle is $\frac{7}{2}$ cm.

Using the given information, calculate the

i. perimeter of the triangle. (2 Marks)

ii. unknown length denoted by x. (1 Mark)

iii. circum-radius for the triangle ABC. (2 Marks)

(ATTEMPT EITHER ANY TWO OF a, b AND c FOR Q.7.)

b. Show that

i. $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{1 + \tan\theta}{1 - \tan\theta}$. (2 Marks)

ii. $\frac{\sin\theta}{\sin 2\theta} + \frac{\sin 2\theta}{\cos\theta} = \frac{1 + 2\sin 2\theta}{2\cos\theta}$. (3 Marks)

(ATTEMPT EITHER ANY TWO OF a, b AND c FOR Q.7.)

c. A terminal ray in the first quadrant makes an angle θ with the *x*-axis.

If $\sin\theta = \frac{7}{12}$, then find

i. $\cos\theta$. (2 Marks)

ii. $\cos 2\theta$. (2 Marks)

iii. $\tan\theta$. (1 Mark)

Q.8. (Total 6 Marks)

i. A function is given as $y = \sin\frac{x}{2}$, where $-\pi \le x \le \pi$.

I. Complete the given table for the function. (2 Marks)

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{2\pi}{4}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{2\pi}{4}$	$\frac{3\pi}{4}$	π
$\sin\frac{x}{2}$									

II. Draw the graph of the function. (2 Marks)

ii. Prove that the function $g(x) = \sin^2 x(a\cos x + b)$ is an even function. (2 Marks)

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END OF PAPER

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