AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XI EXAMINATION

APRIL/ MAY 2019

Mathematics Paper II

Time: 2 hours   Marks: 60

INSTRUCTIONS

Please read the following instructions carefully.

1. Check your name and school information. Sign if it is accurate.

I agree that this is my name and school. **Candidate's Signature**

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2. There are EIGHT questions. Answer ALL questions. Choices are specified inside the paper.

3. When answering the questions:

   Read each question carefully.
   Use a black pointer to write your answers. DO NOT write your answers in pencil.
   Use a black pencil for diagrams. DO NOT use coloured pencils.
   DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
   Complete your answer in the allocated space only. DO NOT write outside the answer box.

4. The marks for the questions are shown in brackets ( ).

5. You may use a scientific calculator if you wish.

Q.1. (Total 4 Marks)

A pair of simultaneous linear equations with complex coefficients is given as

$i\,x-\sqrt{-4}\,y=\frac{8}{i}$ and $\frac{x}{i}-i\,y=i^{3}$. Show that the value of y is real.

Q.2. (Total 7 Marks)

a.

i. Under what condition(s) the given system of linear equations will become homogeneous? (1 Mark)

$$k_{11}y_1 + k_{12}y_2 + k_{13}y_3 - m = 0$$
$$k_{21}y_1 + k_{22}y_2 + k_{23}y_3 - n = 0$$
$$k_{31}y_1 + k_{32}y_2 + k_{33}y_3 - p = 0$$

ii. It is given that $M = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ and I is an identity matrix of order 3.

Find the value of constant λ (lambda) when $M - \lambda I$ is a null matrix. (2 Marks)

b. It is given that $Q^t P^t = \begin{bmatrix} m & 2 & 0 \\ 3 & 0 & 1 \\ 0 & 4 & 10 \end{bmatrix}$.

i. State the matrix PQ. (2 Marks)

ii. Hence, determine the value of m for which the matrix PQ will become singular? (2 Marks)

Q.3. (Total 9 Marks)

a. The arithmetic progression (A.P) and geometric progression (G.P) are defined in such a way that the first term, fourth term and ninth term of an A.P will form a G.P.

i. Express, in terms of a and d, the first, fourth and ninth terms of A.P. (1 Mark)

ii. Use the given condition to show that $2a = 9d$. (3 Marks)

iii. Hence, find the value of tenth term of an A.P. when $a = 4$. (1 Mark)

b. The sum of first n natural numbers $\sum_{k=1}^{n} k$ and the sum of the cubes of first n natural numbers $\sum_{k=1}^{n} k^3$ differ by zero. Find the value of n. (4 Marks)

(**Formulae**: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$)

Q.4. (Total 6 Marks)

Prove that $\binom{n}{3}+\binom{n}{2}=\binom{n+1}{3}$, where n is a positive integer.

Q.5. (Total 7 Marks)

(**Formula**: $T_{r+1} = {}^{n}C_{r}a^{n-r}b^{r}$, where, symbols have their usual meanings.)

In the binomial expansion of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^{n}$,

a. show that $r = \frac{n}{2}$ for which the term is constant. (3 Marks)

b. hence, find the value of n and of k if the constant term is given by ${}^{k}C_{3}$. (4 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.6.)

Q.6. (Total 7 Marks)

a. The length *x* of one side of a square is decreased by 8 cm and another by 4 cm to form a rectangle.

i. Find an expression, in terms of *x*, for

I. the length of the sides of the rectangle. (2 Marks)

II. the area of the rectangle. (1 Mark)

ii. Show that $x^2 + 3x - 8 = 0$, if the square has an area $\frac{1}{5}$ times the area of the rectangle.

(2 Marks)

iii. Without using a calculator, express the length of the side of square in the form $\frac{a+\sqrt{41}}{b}$.

Hence, state the values of *a* and *b*. (2 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.6.)

b. Given that $p=\frac{4}{\omega}, q=-\frac{2}{\omega^2}$ and $r=\frac{1}{\omega^9}$, show that

(**Note**: ω and ω^2 are the complex cube roots of unity.)

i. $p-2q+r^2$ is a real number. (3 Marks)

ii. $\sqrt{p^2+(2q)^2}$ is an imaginary number. (4 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

Q.7. (Total 14 Marks)

a.

i. If $p=\sqrt{\text{cosec}\,A-1}$, $q^2=1+\frac{1}{\sin A}$ and $r=\tan A$, then show that pqr is constant.

(**Note**: p, q and r are positive.) (4 Marks)

ii. If $A+B+C=180^{\circ}$, then show that $\sin(B+C)=\sin A$. (3 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

(**Formulae**: $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$, $2\cos A\cos B = \cos(A+B) + \cos(A-B)$)

b. Without using a calculator, evaluate the following:

i. $\dfrac{\sin 150^{o} - \sin 30^{o}}{\sin^{2} 75^{o} + \cos^{2} 75^{o}}$ (2 Marks)

ii. $(\cos 60^{o} + \cos 20^{o}) \times \cos 80^{o}$ (5 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

c.

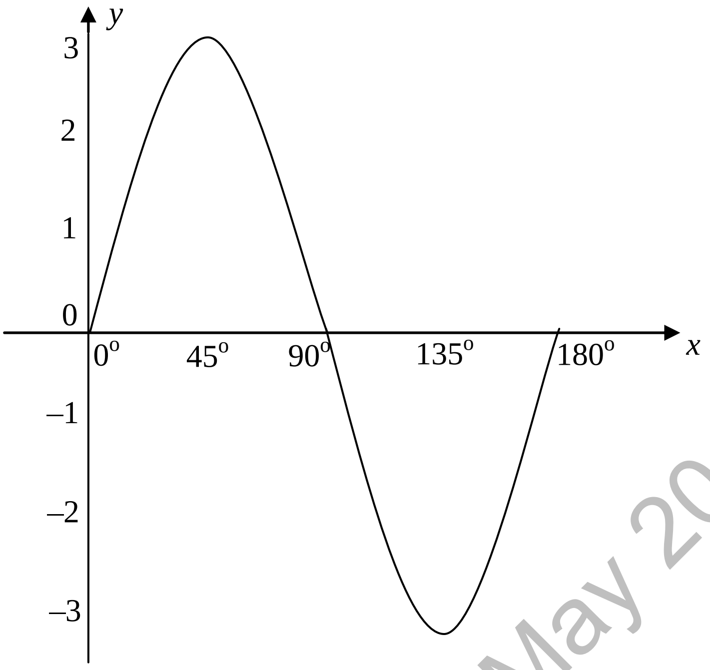
i. Express $\frac{s(s-a)(s-b)(s-c)}{(a+b+c)(abc)}$ in terms of r and R. (4 Marks)

(**Note**: Symbols have their usual meanings.)

ii. A circle is inscribed in a triangle having radius r units. It is given that the area of a circle and perimeter of a triangle are 81π square units and 48 units respectively.
Use $r=\frac{\Delta}{s}$ to find Δ. (3 Marks)

Q.8. (Total 6 Marks)

a. The figure shows the graph of $y = a\sin(px)$ for $0^{\circ} \leq x \leq 180^{\circ}$, where a and p are positive constants.



Use the graph to state the

i. value of a and of p. (2 Marks)

ii. the period of y. (1 Mark)

b. Find the general solution of $\cos x - \sqrt{3}\sin x = 0$. (3 Marks)

END OF PAPER

Please use this page for rough work